DOCUMENT RESUME

ED 190 395 SE 031 457

TITLE	Mathematics, Book S. Teacher Guide.
INSTITUTION	Southwest Educational Development Corp., Austin, Tex.
SPONS AGENCY	Bureau of Elementary and Secondary Education (DHEW/OE), Washington, D.C.: National Science Foundation, Washington, D.C.: Texas Education Agency, Austin.
PUB DATE	70
NOTE	73p.: For related documents, see SE 031 452-456. Contains occasional light and broken type.
EDRS PRICE	MF01/PC03 Plus Postage.
DESCRIPTORS	*Geometric Concepts: Grade 7: Instructional Materials: *Learning Activities: Low Achievement: Mathematical Applications: *Mathematical Concepts: Mathematics Education: Mathematics Instruction: Prime Numbers: Resource Materials: Secondary Education: *Secondary School mathematics: *Teaching Methods
IDENTIFIERS	*Factoring (Mathematics): *Number Operations

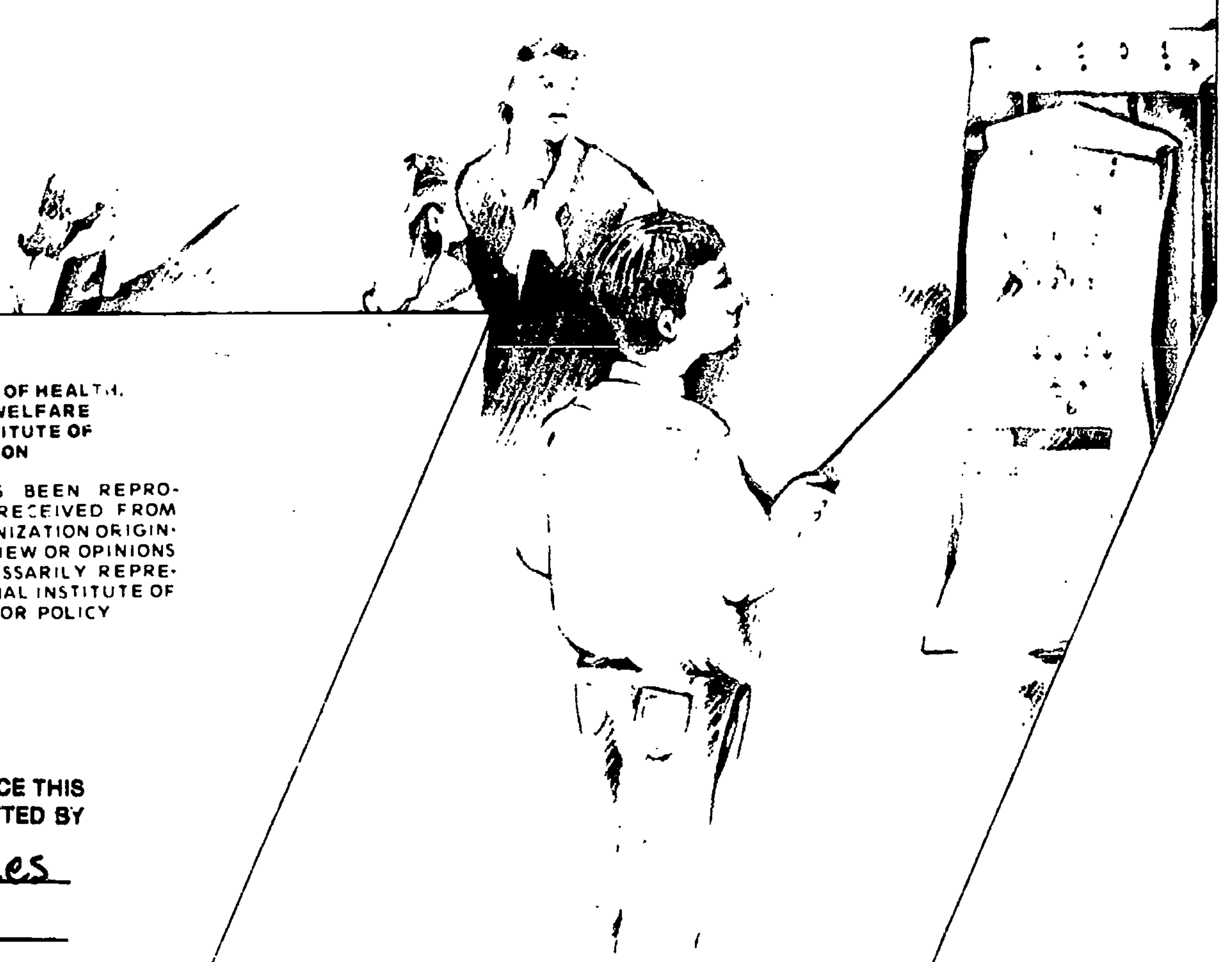
ABSTRACT

This book is the teacher's manual to a text designed to meet the particular needs of those children who have "accumulated discouragements" in learning mathematics. It is a manual of suggested teaching strategies and additional materials aimed at compensating for past student failures to understand mathematical concepts. This book, S, is designed for the common seventh-grade mathematics program. Individual chapter titles are: The Basic Operations: A Different Look: Geometry: and Factors and Primes. (MP)

Southwest Educational Development Laboratory

MATHEMATICS Book S

Teacher Guide



MATHEMATICS
BOOK S

TEACHER GUIDE

SOUTHWEST EDUCATIONAL DEVELOPMENT LABORATORY

These instructional materials were developed and published by the Southwest Educational Development Laboratory using funds from:

a grant from the National Science Foundation,

a contract with the U. S. Office of Education Department of Health, Education, and Welfare, Title IV, Elementary and Secondary Education Act

and,

a contract with the Texas Education Agency.

PREFACE

The teaching strategies and mathematics materials suggested in this teacher's manual and the accompanying mathematics books for children are part of the Southwest Educational Development Laboratory's Mathematics/Science Program.

Users of these adapted materials have the opportunity to revise and improve them in the light of experience and evaluation of results of their effectiveness in the classroom. This interaction of program designers and writers with teachers and pupils is consistent with the process of educational development -- the continuous improvement of materials and techniques. As these materials are pilot tested, the teachers' experiences with them have almost instant impact on their continuing revision and improvement.

Designed to compensate for pupils' past failures to understand mathematical concepts, the Southwest Educational Development Laboratory's Mathematics/Science Education Program takes into account the social and cultural background and cognitive skills a student brings to the learning situation.

This book, Mathematics, Book S, includes adaptations of the mathematics program commonly experienced in the seventh year of school. It is one of three books, R, S, and T, for this level. The adaptations are designed to meet the particular needs of those children who have accumulated discouragements in learning mathematics. With this in mind, the reading level required of the pupils has been reduced. More importantly, meaningful mathematical experiences are presented in ways which give the pupil many opportunities for success.

As in any sound educational program, the role of the teacher is critical. A teacher's interest and enthusiasm are contagious to students, but interest and enthusiasm are dependent upon the teacher's assessment of his own competence. This guide is designed to assist the teacher in directing classroom activity and in developing an understanding and appreciation of the mathematical concepts and skills to be taught.

The following premises guided the team of teachers and mathematicians who adapted and wrote these materials:

- Unnecessary use of vocabulary which has no meaning for children can be avoided.
- Teaching mathematics requires patience, purposeful planning, and opportunity for learning.
- Mathematical experiences can be adapted to children rather than adapting children to mathematical experiences.

The Laboratory's Mathematics Program has been expanded to include Science. Long range plans include adapting science materials to meet the needs of pupils who have failed to respond to traditional materials and teaching approaches.

Edwin Hindsman
Executive Director

ACKNOWLEDGMENTS

These materials were prepared by the Southwest Educational Development Laboratory's Mathematics Education Program during two summer writing conferences. The 1968 Summer Mathematics Writing Conference participated in the initial adaptation of these materials, and the 1969 Summer Mathematics Writing Conference participated in their revision.

The 1969 Summer Mathematics Writing Conference, held in Austin, Texas, was coordinated by Floyd Vest, Professor of Mathematics Education, North Texas State University, Denton, Texas. He was assisted by James Hodge, Professor of Mathematics, North Texas State University, and Palma Lynn Ross, Department of Mathematics, University of Texas at El Paso.

Participants for the 1969 writing conference included: Carmen Montes, Santiago Peregrino, Rebecca Rankin, Rudolph Sanchez, and Flora Ann Sanford, El Paso Independent School District, El Paso, Texas; Jimmye Blackmon, J. Leslie Fauntleroy, Barbara Graham, and Sophie Louise White, East Baton Rouge Parish Schools, Baton Rouge, Louisiana; Lawrence A. Couvillon and James Keisler, Louisiana State University, Baton Rouge, Louisiana; and Socorro Lujan, Mathematics Education, Southwest Educational Development Laboratory, Austin, Texas.

Consultants for this conference included: Sam Adams, Louisiana State University, Baton Rouge, Louisiana; James Anderson, New Mexico State University, Las Cruces, New Mexico; R. D. Anderson, Louisiana State University, Baton Rouge, Louisiana; Robert Cranford, North Texas State University, Denton, Texas; William T. Guy, Jr., University of Texas at Austin, Austin, Texas;

Lenore John, School Mathematics Study Group, Stanford, California; Houston T. Karnes, Louisiana State University, Baton Rouge, Louisiana; and B. G. Nunley North Texas State University, Denton, Texas

The 1968 Summer Mathematics Writing Conference was coordinated by James Keisler, Professor of Mathematics, Louisiana State University. Participants for this conference included: Stanley E. Ball, University of Texas at El Paso, El Paso, Texas; Lawrence A. Couvillon, Louisiana State University, Baton Rouge, Louisiana; Rosalie Espy, Alamo Heights Independent School District, San Antonio, Texas; J. Leslie Fauntleroy, East Baton Rouge Parish Schools, Baton Rouge, Louisiana; Norma Hernandez, University of Texas at Austin, Austin, Texas; Glenda Hunt, University of Texas at Austin, Austin, Texas; Carmen Montes, El Paso Independent School District, El Paso, Texas; Santiago Peregrino, El Paso Independent School District, El Paso, Texas; Rebecca Rankin, El Paso Independent School District, El Paso, Texas; Ida Slaughter, East Baton Rouge Parish Schools, Baton Rouge, Louisiana; and Sister Gloria Ann Fielder, CDP, Our Lady of the Lake College, San Antonio, Texas.

Consultants for this conference included: R. D. Anderson, Louisiana State University, Baton Rouge, Louisiana; William DeVenney, School Mathematics Study Group, Stanford, California; Sister Claude Marie Faust, Incarnate Word College, San Antonio, Texas; Mary Folsom, University of Miami, Coral Gables, Florida; William T. Guy, Jr., University of Texas at Austin, Austin, Texas; Houston T. Karnes, Louisiana State University, Baton Rouge, Louisiana; William McNabb, St. Marks School, Dallas, Texas; Sheldon Myers, Educational Testing Service, Princeton, New Jersey; and Ann Tinsley, East Baton Rouge Parish Schools, Baton Rouge, Louisiana.

Acknowledgment also is given to the Educational Development Center, Newton, Massachusetts, and to the School Mathematics Study Group, Stanford, California, for their permission to use materials from their programs as a foundation for adaptation and development.

Mathematics Education Program Staff

Rex Arnett, Program Director
E. Glenadine Gibb, Program Director
1968-69
Thomas H. Scannicchio, Coordinator

TABLE OF CONTENTS

Chapter 5:

The Basic Operations: A Different Look

Teacher Commentary

There are numerous exercises in this chapter. Many times two, or perhaps even three exercises can be covered the same day. The material is divided into short units so that it is possible to omit some, if class performance so dictates, without affecting continuity.

General Objectives

A. To reinforce the students' skills in computation.

B. To show how the algorithms (standard computational processes) for the basic operations were developed and to point out the efficiency gained therefrom.

C. To provide the students with a means of graphically illustrating the basic operations by making use of the number line.

Behavioral Objectives:

1. Using expanded form, the student can find the sum of two numbers (each named by less than four digits).

2. The student can find the sum of two numbers (each named by less than five digits) using partial sums.

3. The student can find the sum of up to three numbers (each named by less than four digits) using the standard addition algorithm.

4. The student can draw a number line illustration of the addition of two numbers (each named by one digit).

5. Given a number line illustration of the addition of two numbers (each named by one digit), the student can identify the problem being illustrated.

6. Given a subtraction problem such as <u>764 - 199</u>, the student can illustrate the process of "carrying" (regrouping) by using expanded form to perform the operation.

7. The student can subtract two numbers (each named by less than four digits) using the standard subtraction algorithm.

8. The student can draw a number line illustration of the subtraction of two numbers (each named by one digit).

9. Given a number line illustration of the subtraction of two numbers (each named by one digit), the student can identify the problem being illustrated.

10. Given a multiplication problem such as <u>2,317 · 10,000</u> (both multiplicand and multiplier named by less than five digits, and one factor a multiple of ten), the student can state, for example, that <u>the product is 2,317, followed by four zeros</u> (23,170,000).

11. The student can write a number named by less than seven digits (the last digit being a zero) as a product of two numbers, one of which is a power of ten. For example: <u>13,500 = 135 · 100</u>.

12. Given two multiples of ten (each named by less than five digits) such as 1,200 • 3,000, the student can state, for example, that the product is 12 • 3, followed by five zeros (3,600,000).
13. Using expanded form, the student can find the product of two numbers (each named by less than three digits).
14. Using partial products, the student can find the product of two numbers (each named by less than four digits).
15. The student can draw a number line illustration of the multiplication of two numbers (each named by one digit).
16. The student can find the quotient of a problem such as 115 ÷ 16 using the process of repeated subtraction.
17. The student can divide a number named by less than five digits by a number named by less than four digits, using the standard division algorithm.

Section 5-1 Addition in Expanded Form

"Expanded form" here and throughout the chapter refers to a number being written according to how it is read. Thus, 3,496 is read "three thousand, four hundred, ninety-six" and is written in expanded form as 3,496 = 3000 + 400 + 90 + 6.

Answers to Exercise 5-1

Student Text - Page 2

1.	385	5.	1,252
2.	1,710	6.	1,841
3.	1,741	7.	12,222,221
4.	1,110	8.	Twelve million, two hundred twenty-two thousand, two hundred twenty-one

Section 5-2 Addition in Short Form

Make sure all students know the meaning of digits and partial sum.

Answers to Exercise 5-2

Student Text - Page 3

1.	63	6.	1,928
2.	57	7.	2,375
3.	79	8.	15,369
4.	1,051	9.	167,899
5.	8,219	10.	11,110

Answers to Exercise 5-2(a)

Student Text - Page 5

1.	1 ten	7.	1,119
2.	1 hundred	8.	1,701
3.	hundred	9.	18,268
4.	94	10.	244,444,442
5.	148	11.	two hundred forty-four million, four hundred forty-four thousand, four hundred forty-two
6.	143		

Section 5-3 The Number Line

Stress that the distance between 0 and 1 on the number line can be any distance. But once a unit distance is picked, that will be the distance between any two consecutive whole numbers on the number line.

Answers to Exercise 5-3

Student Text - Page 6

1.	0	4.	a and b. a and b are any two consecutive whole numbers.
2.	1	5.	the line continues infinitely in either direction.
3.	same	6.	No; given any number n on the number line, there will always be an n + 1.

Answers to Exercise 5-4

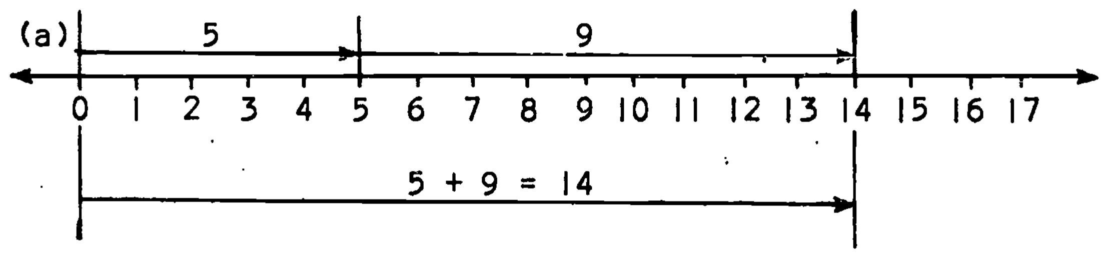
Student Text - Page 7

1. 7 + 5 = 12

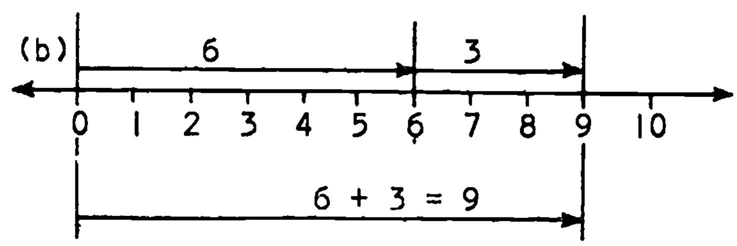
2. 3 + 5 = 8

3. 5 + 4 = 9

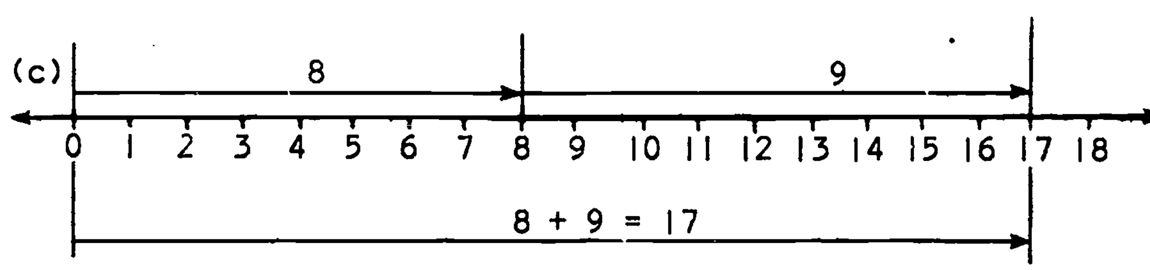
4. (a)



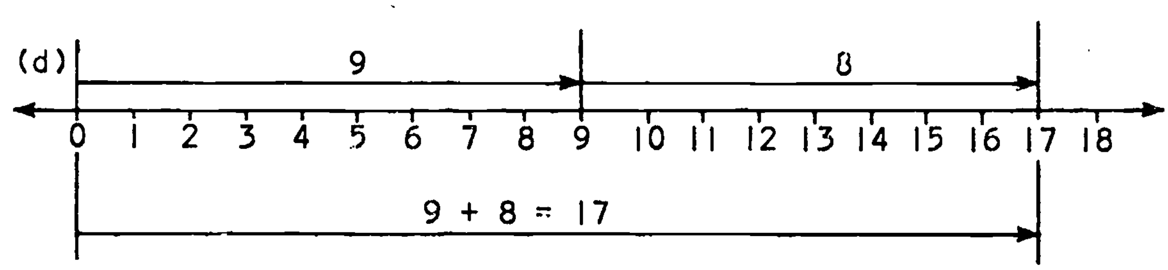
(b)



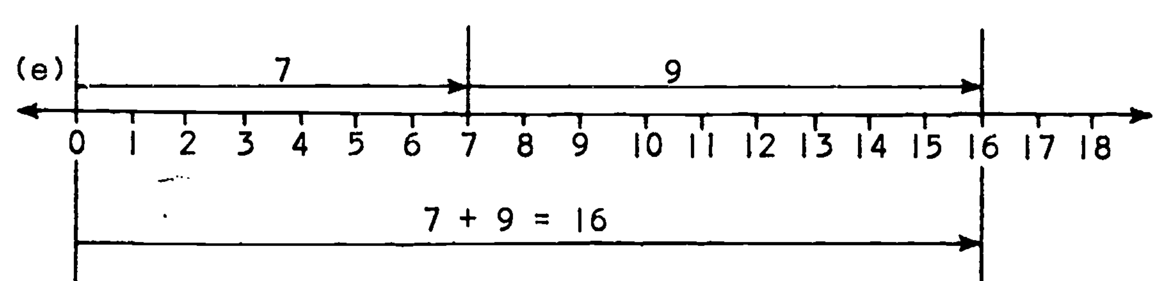
(c)



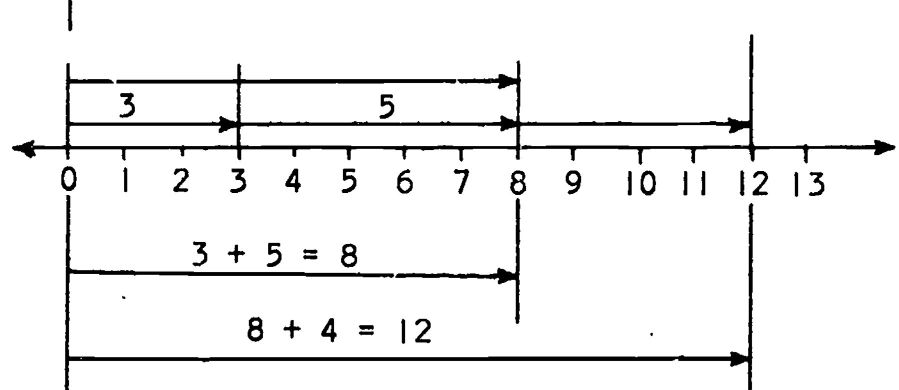
(d)



(e)



5.



Section 5-5 Regrouping in Subtraction

Make sure all students know the meaning of the words compute and regroup.

Answers to Exercise 5-5

Student Text - Page 9

1. 17
2. 38
3. 565
4. 263
5. 423
6. 1,889

Answers to Exercise 5-6

Student Text - Page 11

1. 19
2. 17
3. 88
4. 284
5. 256
6. 1,878

Answers to Exercise 5-7

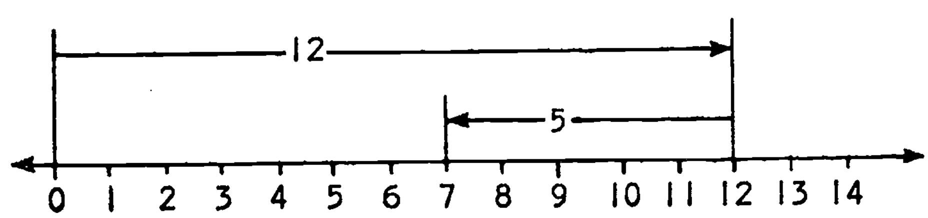
Student Text - Page 13

1. 107
2. 142
3. 187
4. 27
5. 1,336
6. 280
7. 57
8. 132
9. 1,332
10. 1,127

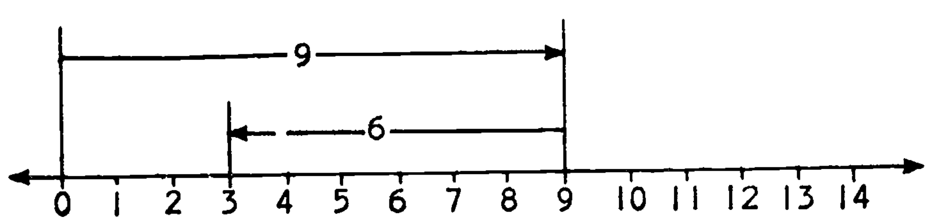
Answers to Exercise 5-8

Student Text - Page 14

1.

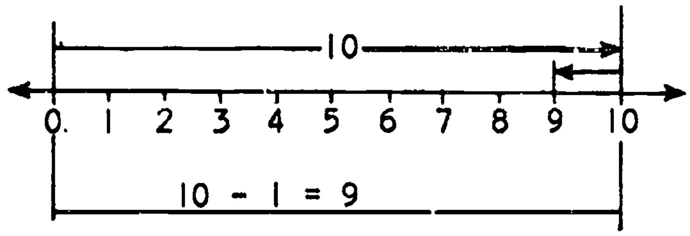


2.

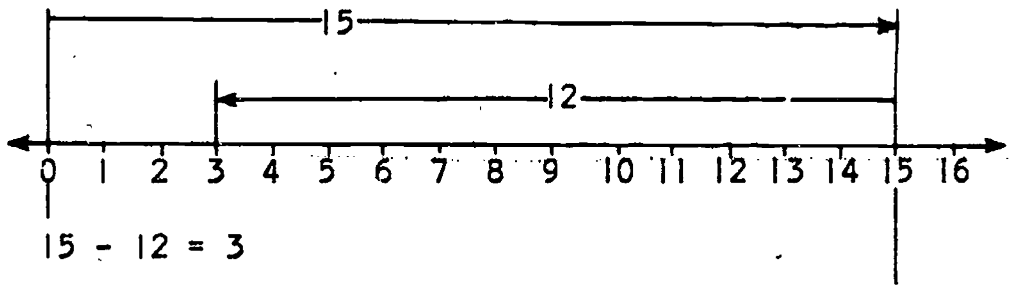


9 - 6 = 3

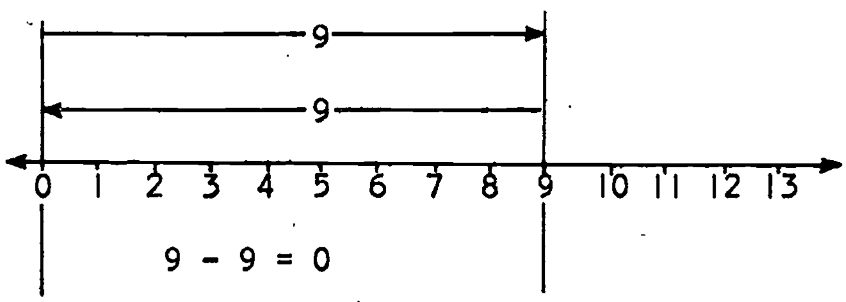
3.



4.



5.



6. 9 - 6 = 3

Section 5-9 Multiplication with Numbers Ending in Zero

Point out the different names of the parts of a multiplication problem -- multiplicand, multiplier, and product. But use these words only when they facilitate reference to a problem. You will note that the word multiplicand does not appear in the student text.

The commutative and associative properties of multiplication are introduced here, indirectly. There is no need at this point to introduce the properties by name or to state them in their general form. Do stress the need for (and careful use of) parentheses.

Answers to Exercise 5-9

Student Text - Page 15

1. 70 2. 500

3. 80,000 4. 32,000

5. 6,900,000
6. 5,070
7. 17,280,000
8. 4,000,000
9. 9,800
10. 320
11. $36 \cdot 10$
12. $58 \cdot 100$
13. $9 \cdot 10$
14. $397 \cdot 1000$
15. $25 \cdot 10{,}000$
16. $97 \cdot 100$
17. $546 \cdot 1{,}000{,}000$
18. $705 \cdot 100$
19. $87 \cdot 100$
20. $101 \cdot 10$

<u>Answers to Exercise 5-9(a)</u>

Student Text - Page 16

1. Yes
2. Yes
3. Yes
4. Yes
5. Yes
6. 21
7. 100
8. Yes
9. 2,100
10. $p = 30 \cdot 40$
$p = (3 \cdot 10) \cdot (4 \cdot 10)$
$p = 3 \cdot 4 \cdot 10 \cdot 10$
$p = 12 \cdot 100$
$p = 1{,}200$
11. $p = 70 \cdot 800$
$p = (7 \cdot 10) \cdot (8 \cdot 100)$
$p = 7 \cdot 8 \cdot 10 \cdot 10 \cdot 10$
$p = 56 \cdot 1000$
$p = 56{,}000$
12. $p = 900 \cdot 60$
$p = (9 \cdot 100) \cdot (6 \cdot 10)$
$p = 9 \cdot 6 \cdot 100 \cdot 10$
$p = 54 \cdot 1000$
$p = 54{,}000$
13. $p = 9 \cdot 400$
$p = 9 \cdot (4 \cdot 100)$
$p = 36 \cdot 100$
$p = 3{,}600$

14.
$p = 600 \cdot 800$
$p = (6 \cdot 100) \cdot (8 \cdot 100)$
$p = 6 \cdot 8 \cdot 100 \cdot 100$
$p = 48 \cdot 10{,}000$
$p = 480{,}000$

15.
$p = 7{,}000 \cdot 500$
$p = (7 \cdot 1000) \cdot (5 \cdot 100)$
$p = 7 \cdot 5 \cdot 1000 \cdot 100$
$p = 35 \cdot 100{,}000$
$p = 3{,}500{,}000$

Answers to Exercise 5-9(b)

Student Text - Page 19

1. 15,000
2. 35,000
3. 2,700,000
4. 72,000
5. 540,000
6. 560,000
7. 490,000
8. 360,000
9. 81,000,000
10. 2,800,000

Section 5-10 Multiplication in Expanded Form

The distributive property of multiplication with respect to addition is being introduced here indirectly. There is no need at this point to introduce the property by name or to state it in its general form.

Answers to Exercise 5-10

Student Text - Page 20

1. 188
2. 612
3. 441
4. 205
5. 784
6. 1,584
7. 3,311
8. 2,961

Answers to Exercise 5-10(a)

Student Text - Page 22

1. 188
2. 612
3. 441
4. 205
5. 2,576
6. 1,584
7. 3,784
8. 4,935

Answers to Exercise 5-10(b)

Student Text - Page 23

1. 1,608
2. 2,835
3. 6,693
4. 2,496
5. 23,182
6. 11,352
7. 40,272
8. 68,355
9. 243,000

Section 5-11 Multiplication in Short Form

Make sure all students know the meaning of partial product.

Answers to Exercise 5-11

Student Text - Page 26

1. 2 tens or 20
2. 4 times 40 is 160
3. 18 tens or 180
4. 3 tens or 30
5. 8 tens or 80
6. 30; 40; 1200
7. 1380
8. $\begin{array}{r} 678 \\ \times\underline{439} \end{array}$

$$\begin{array}{rl} 72 & = (9 \cdot 8) \\ 630 & = (9 \cdot 70) \\ 5400 & = (9 \cdot 600) \\ 240 & = (30 \cdot 8) \\ 2100 & = (30 \cdot 70) \\ 18000 & = (30 \cdot 600) \\ 3200 & = (400 \cdot 8) \\ 28000 & = (400 \cdot 70) \\ \underline{240000} & = (400 \cdot 600) \end{array}$$

297,642

Answers to Exercise 5-11(a)

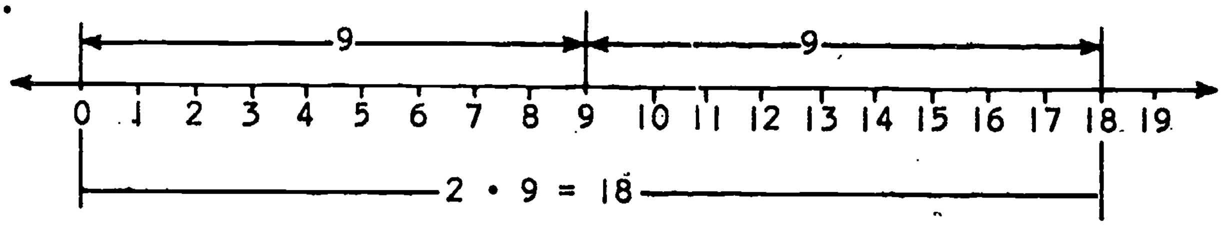
Student Text - Page 27

1. 486
2. 512
3. 3,346
4. 6,680
5. 44,988
6. 4,368
7. 5,152
8. 2,958
9. 39,812
10. 314,712

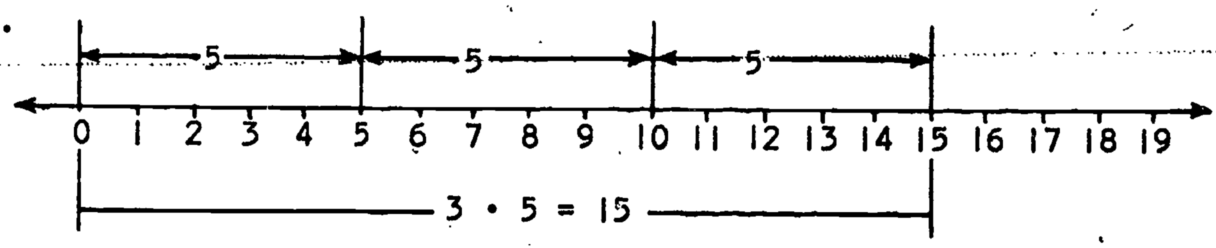
Answers to Exercise 5-12

Student Text - Page 28

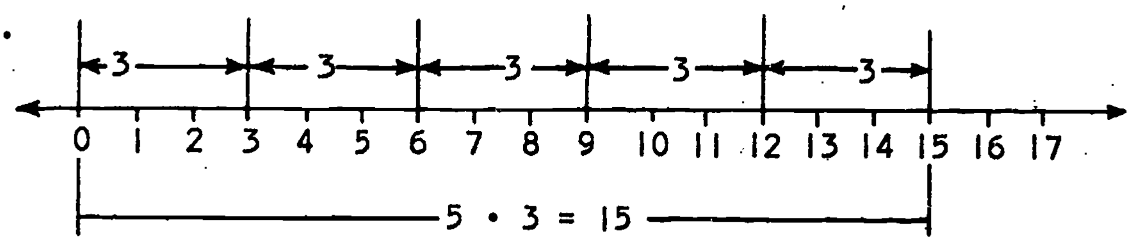
1.



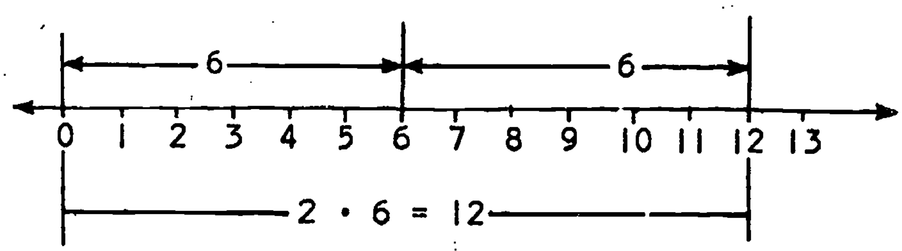
2.



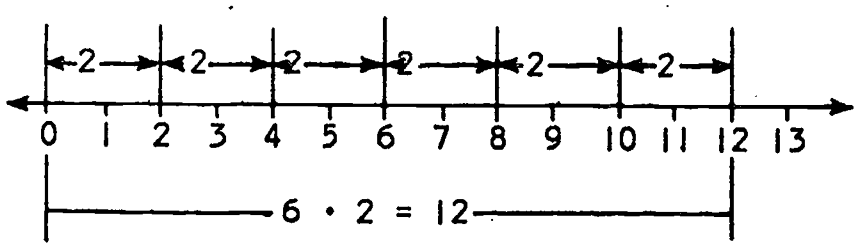
3.



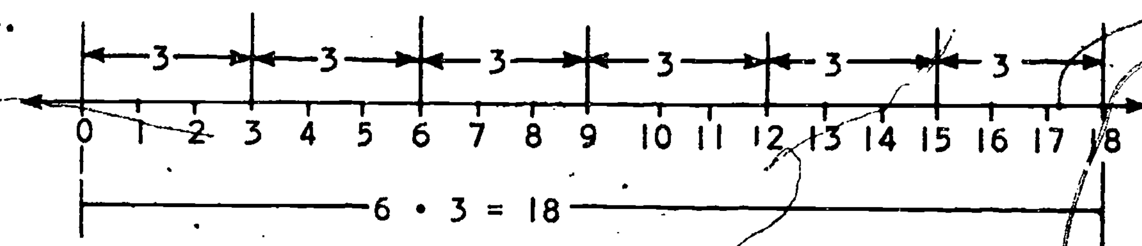
4.



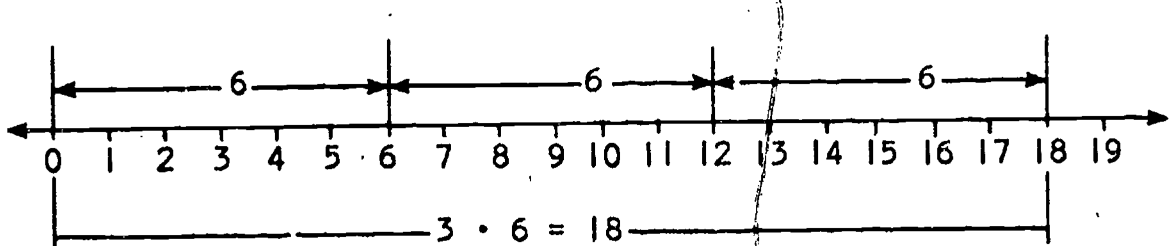
5.



6.



7.



Section 5-13 Division As Repeated Subtraction

Point out the different names of the parts of a division problem -- divisor, dividend, and quotient. But use these words only when they facilitate reference to a problem.

Answers to Exercise 5-13

Student Text - Page 30

1. $$\begin{array}{r} 63 \\ -\underline{7} \\ 56 \\ -\underline{7} \\ 49 \\ -\underline{7} \\ 42 \\ -\underline{7} \\ 35 \\ -\underline{7} \\ 28 \\ -\underline{7} \\ 21 \\ -\underline{7} \\ 14 \\ -\underline{7} \\ 7 \\ -\underline{7} \\ 0 \end{array}$$

2. $$\begin{array}{r} 125 \\ -\underline{25} \\ 100 \\ -\underline{25} \\ 75 \\ -\underline{25} \\ 50 \\ -\underline{25} \\ 25 \\ -\underline{25} \\ 0 \end{array}$$

3. $$\begin{array}{r} 72 \\ -\underline{12} \\ 60 \\ -\underline{12} \\ 48 \\ -\underline{12} \\ 36 \\ -\underline{12} \\ 24 \\ -\underline{12} \\ 12 \\ -\underline{12} \\ 0 \end{array}$$

4. $$\begin{array}{r} 66 \\ -\underline{13} \\ 53 \\ -\underline{13} \\ 40 \\ -\underline{13} \\ 27 \\ -\underline{13} \\ 14 \\ -\underline{13} \\ r \rightarrow 1 \end{array}$$

5. $$\begin{array}{r} 38 \\ -\underline{3} \\ 35 \\ -\underline{3} \\ 32 \\ -\underline{3} \\ 29 \\ -\underline{3} \\ 26 \\ -\underline{3} \\ 23 \\ -\underline{3} \\ 20 \\ -\underline{3} \\ 17 \\ -\underline{3} \\ 14 \\ -\underline{3} \\ 11 \\ -\underline{3} \\ 8 \\ -\underline{3} \\ 5 \\ -\underline{3} \\ r \rightarrow 2 \end{array}$$

Answers to Exercise 5-14

Student Text - Page 31

1. 19, r = 0
2. 24, r = 1
3. 15, r = 3
4. 131, r = 3
5. 222, r = 2
6. 157, r = 4
7. 1321, r = 3
8. 1404, r = 3
9. 2358, r = 3

Answers to Exercise 5-14(a)

Student Text - Page 33

1. 20, r = 34
2. 71, r = 7
3. 14, r = 53
4. 23, r = 34
5. 186, r = 26
6. 1188, r = 21

Answers to Exercise 5-14(b)

Student Text - Page 35

1. 137
2. 1,210, r = 3
3. 6,636, r = 3
4. 1,140, r = 3
5. 54, r = 8
6. 20, r = 34

Answers to Exercise 5-14(c)

Student Text - Page 36

1. 321
2. 212
3. 166, r = 1
4. 162
5. 813, r = 4
6. 862
7. 821, r = 2
8. 724, r = 8
9. 991, r = 4

Answers to Exercise 5-14(d)

Student Text - Page 39

1. 7, r = 30
2. 20, r = 34
3. 71, r = 7
4. 129, r = 61
5. 461, r = 6
6. 245
7. 132
8. 121, r = 55
9. 989

Answers to Review Exercise

Student Text - Page 40

1. (a) digits - the set of symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
   (b) partial sum - a sum obtained by adding only part of the numbers in a problem; particularly, the sum of ones, the sum of tens, or the sum of hundreds, etc.
   (c) number line - a line with whole numbers marked on it so that the distance between 0 and 1 is the same as the distance between 1 and 2 or 2 and 3 or 3 and 4, etc.
   (d) regrouping - writing a number as a sum of other groups. Thus, 56 = 5 (10) + 6 (1) = 50 + 6.
   (e) quotient - the result obtained by dividing one number by another.
   (f) partial quotient - one of a series of results obtained by deciding how many times a divisor can be subtracted from a dividend.
   (g) product - the result obtained from the multiplication of two or more numbers.
   (h) difference - the result obtained by subtracting one number from another.
   (i) multiplier - the number by which we multiply another number.
   (j) partial product - a product obtained by multiplying only part of the numbers in a problem; particularly, multiplying ones by ones, ones by tens, tens by tens, tens by hundreds, etc.
   (k) divisor - the number by which we divide another number.

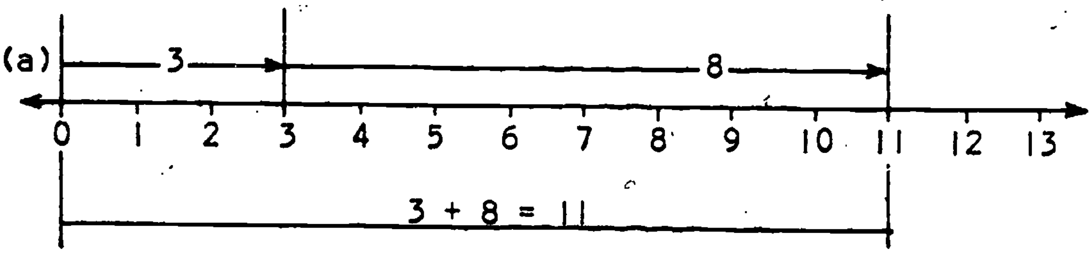
2. (a) 32,706 (e) 9,204

(b) 50,291 (f) 43,249

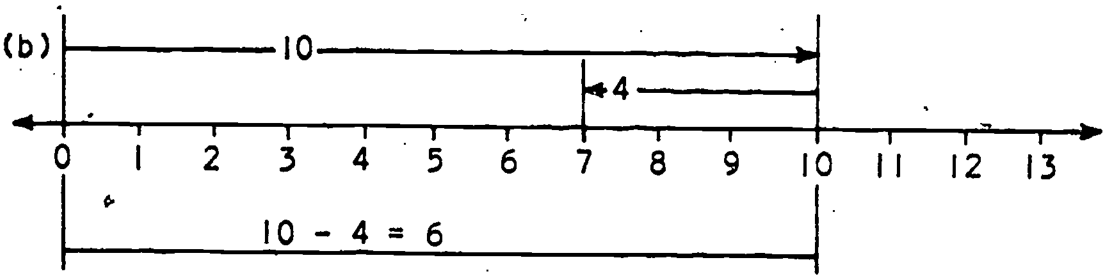
(c) 52,595 (g) 309

(d) 4,239 (h) 408

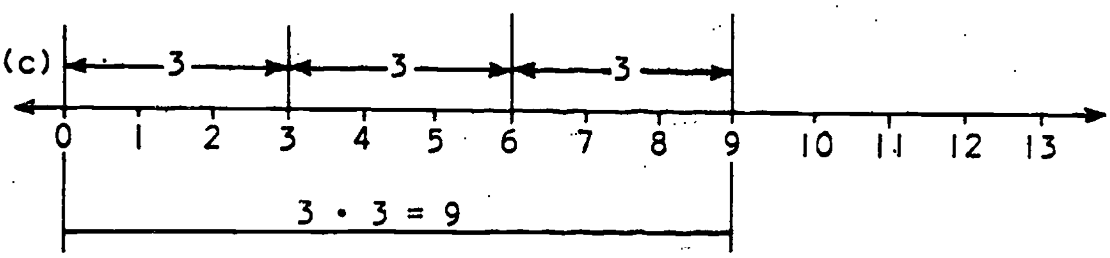
3. (a)



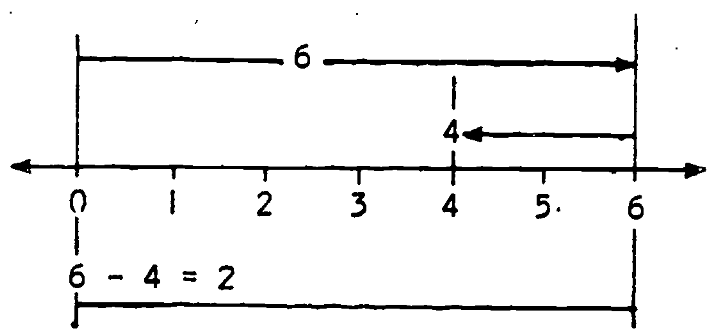
(b)



(c)



4.



Chapter 6:

Geometry

Teacher Commentary

There is a wealth of material in this chapter. It is designed basically for class participation; it is therefore suggested that homework be kept to a minimum. Many of the activities are suitable for group projects as well as for total class involvement.

General Objectives

A. To develop the ideas of point, line, and plane as means of classifying and describing special sets of points which we refer to as geometric figures.

B. To describe how points, lines, and planes relate to each other.

C. To describe the properties of certain geometric figures and show how they relate to the real world.

D. To develop students' skills in constructing pictures of certain geometric figures.

Behavioral Objectives

1. The student can draw a picture of a point and describe its characteristics.

2. The student can draw a picture of each of the following and describe each as a set of points.

(For example: "A line segment is a set of points made up of two endpoints and all the points between them.")

a. line segment

b. ray

c. plane

d. simple open path

e. simple closed path

f. circle

3. The student can draw a picture of each of the following and describe each in terms of <u>rays</u>:

a. line

b. angle

4. The student can draw a picture of a triangle and describe it in terms of line segments or paths.

5. The student can draw a picture of a rectangle and describe it in terms of line segments and square corners.

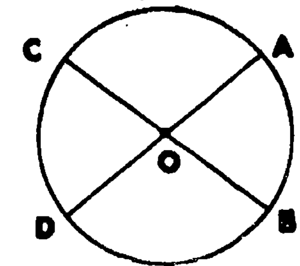
6. Given a picture of a triangle, the student can identify and name the set of points in the interior of the figure. (triangular region)

7. Given a picture of a polygon of no more than five sides, the student can identify and name the vertices and sides.

8. Given a picture of a convex polygon and a concave polygon, the student can identify and distinguish between the two.

9. The student can draw a picture of a four-sided, a five-sided, and a six-sided polygon, and write the special name of each. (quadrilateral, pentagon, hexagon, respectively)

10. Given a picture of a circle marked as the one shown below, the student can identify and name the center, a radius, a diameter, an arc, and a semi-circle.



11. Given two line segments of unequal lengths, the student can, without using a ruler, determine if the line segments fit exactly (are congruent).

12. The student can, on a given ray, mark a line segment congruent to a given line segment.

13. The student can construct the perpendicular bisector of a given line segment, using either a straight edge and a compass or by paper folding.

14. The student can determine the midpoint of a given line segment by paper folding.

15. Given a picture of two intersecting lines, the student can, by use of a paper square corner, determine if the lines are perpendicular.

16. Given a picture of two perpendicular lines, properly labeled, the student can identify and name a right angle.

17. Given a picture of two non-parallel lines, properly labeled, the student can identify and name the point of intersection.

18. Given a picture of two lines in a plane, the student can determine if the lines:

    (a) have no point in common (are parallel), or,

    (b) have one point in common (intersect), or,

    (c) are the same line.

19. Given a picture of a line and a point <u>not</u> on the line, and using a straight edge and a paper square corner (or a straight edge and a compass), the student can construct a line through the given point perpendicular to the given line.
20. Given a picture of a line and a point <u>not</u> on the line, and using a straight edge, the student can construct a line through the given point parallel to the given line.
21. Using a straight edge and a paper square corner, the student can construct a line parallel to a given line.
22. Given a picture of two parallel lines, and using a straight edge and a paper square corner, the student can construct a line perpendicular to both parallel lines.
23. Given a picture of an angle, the student can identify and name the sides and the vertex.
24. Given a picture of a right angle, the student, using a paper square corner, can verify that the angle is a right angle.
25. Given a picture of two angles, the student, without using a protractor, can determine if the angles are congruent.
26. Given a picture of two intersecting lines, the student can identify and name the vertically opposite angles.
27. Given a picture of two parallel lines cut by a transversal, the student can identify and name the congruent alternate interior angles.

28. Given a picture of an angle, the student, using a compass and a straight edge, can construct a copy of the given angle.

29. Given a picture of an angle, the student, using a compass and a straight edge, can construct a line bisecting the angle into two congruent parts.

Section 6-1 Introduction

The analogy that we make here between numbers in arithmetic and points is intended to get the students to realize that numbers and points are not physical objects that we can see or feel.

We invent numbers to help us in classifying sets of objects as to how many objects the sets contain and perform operations on these numbers. We also invent points to help us classify certain geometric figures and to study some of the properties they possess and the relations these figures have to the real world However, you should not over-stress these concepts. Do not be concerned if your students do not readily accept the idea that points exist only in our minds. As they mature mathematically these ideas will become much easier for them to accept.

Section 6-2 Points

The important thing here is for the pupil to recognize that a point denotes position, but has no size. You might draw a picture like this on the blackboard: • ● ● and ask which is a better picture of what we imagine a point to be.

Section 6-3 Line Segments

The important thing in this section is that a line segment is a set of points -- two special points called endpoints and all the points between. To develop the concept of a line the idea of betweenness is introduced here. We say that one point is between two other points if no two of the points are the same and all three points are on the same line.

Using the undefined terms point and between, we can now define a line segment $\overline{AB}$ as the set consisting of points A and B and all points between A and B.

Answers to Exercise 6-3

Student Text - Page 46

1. (a) E is between A and C.
   (b) E is between A and D.
   (c) E is between A and B.
   (d) C is between A and D.
   (e) C is between A and B.
   (f) C is between E and D.
   (g) C is between E and B.
   (h) D is between A and B.
   (i) D is between E and B.
   (j) D is between C and B.

2. (a) Not true.
   (b) Not true.
   (c) True.
   (d) True.

(e) Not true.

(f) Not truo.

(g) True.

(h) Not true.

(i) True.

3. (b) and (d).

4. (a) 5; 3; 4.

(b) D.

(c) B and G. (Each is the endpoint of 6 segments.)

(d) 5. (They are line segments $\overline{AE}$, $\overline{CD}$, $\overline{CF}$, $\overline{DE}$, and $\overline{DF}$.)

(You will recall that this problem, no. 5, is also treated in Section 5 of Chapter 1.)

5. (a) 3 points, not all on the same line segment, determine 3 line segments.

(b) 4 points, no three on the same line segment, determine 6 line segments.

(c) 5 points, no three on the same line segment, determine 10 line segments.

(d) 6 points, no three on the same line segment, determine 15 line segments.

(e) The pupil should see that for each additional point he can draw as many new line segments as he had points before. Thus, if he adds a seventh point, he can draw 6 new line segments, giving him a total of 21.

Section 6-4 Rays and Lines

The important ideas here are that a ray has one endpoint and extends indefinitely in one direction while a line has no endpoints and extends indefinitely in both directions. The idea of infinity may be presented to the student simply as "something that goes on and on forever."

Emphasize that rays and lines are sets of points. We could define ray AB as: the set of points consisting of points A and B, all points P such that P is between A and B, and all points X such that B is between A and X. However, the phrase "Ray AB is the ray from <u>A</u> through <u>B</u>" is easier for the pupil to remember and is suggested very nicely by the symbol for a ray, $\overrightarrow{AB}$.

Be sure that the pupils understand what is meant by <u>opposite rays</u>. We say that $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are <u>opposite rays</u> if A is between B & C.

Answers to Exercise 6-4

Student Text - Page 51

1. The ray from Q through P or ray QP; the ray from M through X or the ray MX; the ray from S through T or $\overrightarrow{ST}$. (Notice that to say "ray $\overrightarrow{ST}$" would be redundant. We read "$\overrightarrow{ST}$" as "ray ST".)

2. (a) $\overleftrightarrow{AB}$; $\overleftrightarrow{BC}$; $\overleftrightarrow{CA}$.

   (b) $\overline{AB}$; $\overline{BC}$; $\overline{CA}$.

   (c) $\overrightarrow{AB}$; $\overrightarrow{BC}$; $\overrightarrow{CA}$; $\overrightarrow{BA}$; $\overrightarrow{CB}$; $\overrightarrow{AC}$.

3. The line segment $\overline{AB}$ or $\overline{BA}$.

4. No; point A is not between point B and point C.

5. No; the points A, B and C are not on the same line.

6. This set defines ray BA.

7. A <u>ray</u> has (a) one endpoint.

   A <u>line</u> has (d) no endpoint.

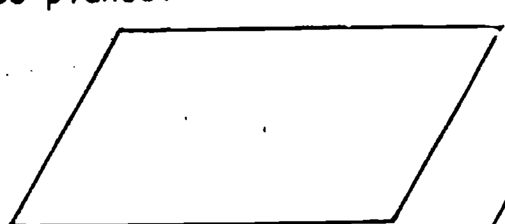
   A <u>line segment</u> has (b) two endpoints.

<u>Section 6-5 Flatness and Planes</u>

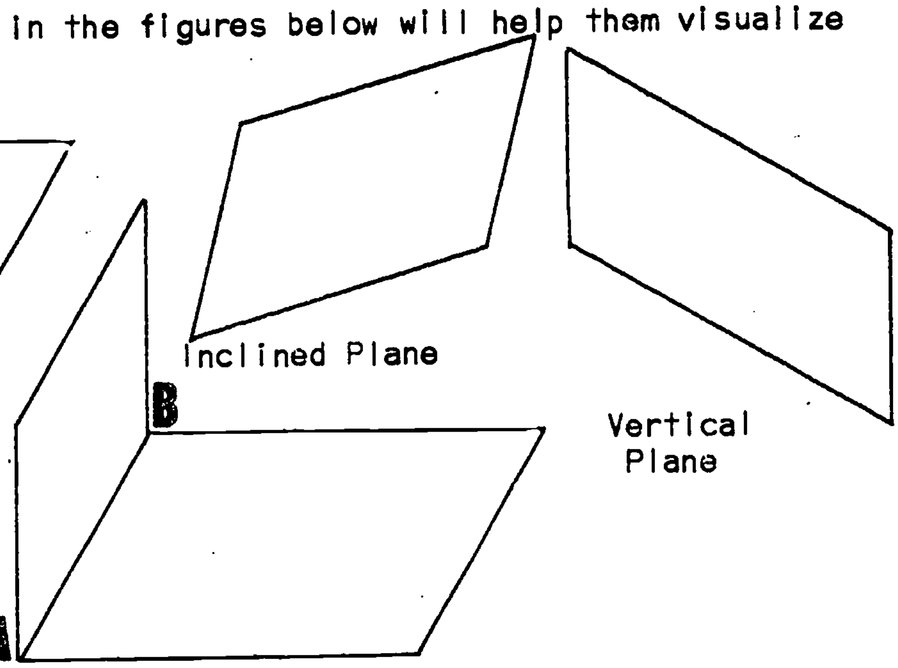
The idea of <u>flatness</u> is important to the pupils' concept of a plane. Emphasize that a surface need not be level to be flat. Most of your pupils will already know what is meant by a flat surface, so that much time need not be spent discussing this concept.

The two important ideas are that the students recognize that any flat surface can represent a plane and that a plane is a set of points.

Some of your pupils may not be able to visualize a plane as easily as they could visualize a ray or a line. You should have them sketch, free-hand, pictures of vertical, horizontal, and inclined planes. A free-hand sketch of a vertical plane and a horizontal plane meeting or intersecting each other as in the figures below will help them visualize these planes.



Horizontal Plane



Vertical and Horizontal Planes Intersecting in line AB

Emphasize that the pictures we draw of planes are necessarily bounded, and hence represent only parts of planes. A plane itself has infinite length and width, but not thickness.

Answers to Exercise 6-5

Student Text - Page 54

1. (a) Yes.

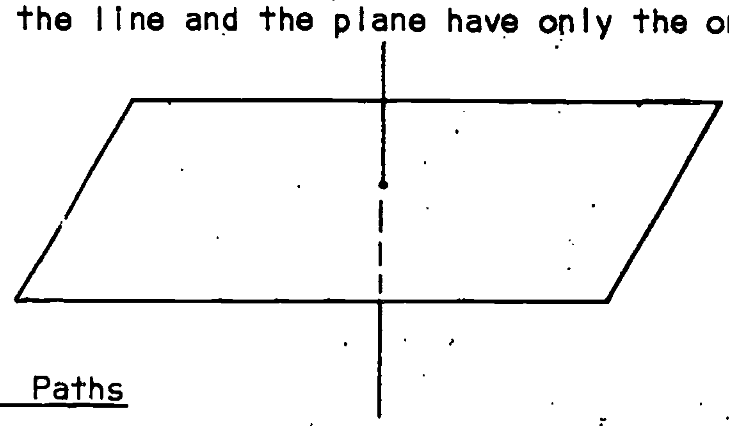
   (b) Yes.

   (c) Yes.

2. One (The three planes have just the corner point in common.)

3. Yes.

4. Yes. (If a line pierces a plane such as a pin pierces a piece of paper, the line and the plane have only the one point in common.)



Section 6-6 Paths

We include this section so that the pupil will know what is meant by a simple open path and a simple closed path. In Section 6-8 we shall be primarily interested in those simple closed paths that are polygons. In this section the student will only be asked to recognize simple closed paths.

Answers to Exercise 6-6

Student Text - Page 56

1. (i) (a), (b), (d), (e), (g), (h)

   (ii) (a), (e), (h)

   (iii) (b), (d), (g)

Section 6-7 Regions

In this section we define a triangle, a triangular region, a rectangle and a rectangular region. Be sure that your pupils understand that a triangle is a set of points and it is the set of points that lie on the three line segments that form the triangle. Too often the pupils think of a triangle as being the set of points that are in the interior of the triangle. Similarly for a rectangle or any other polygon.

If A, B, and C are any three points not on the same line, then we define triangle ABC as the union of the three line segments, $\overline{AB}$, $\overline{AC}$, $\overline{BC}$.

A triangular region is a set of points consisting of the points in the interior of a triangle and the points on the triangle. Similar definitions could be given for a rectangle and rectangular region.

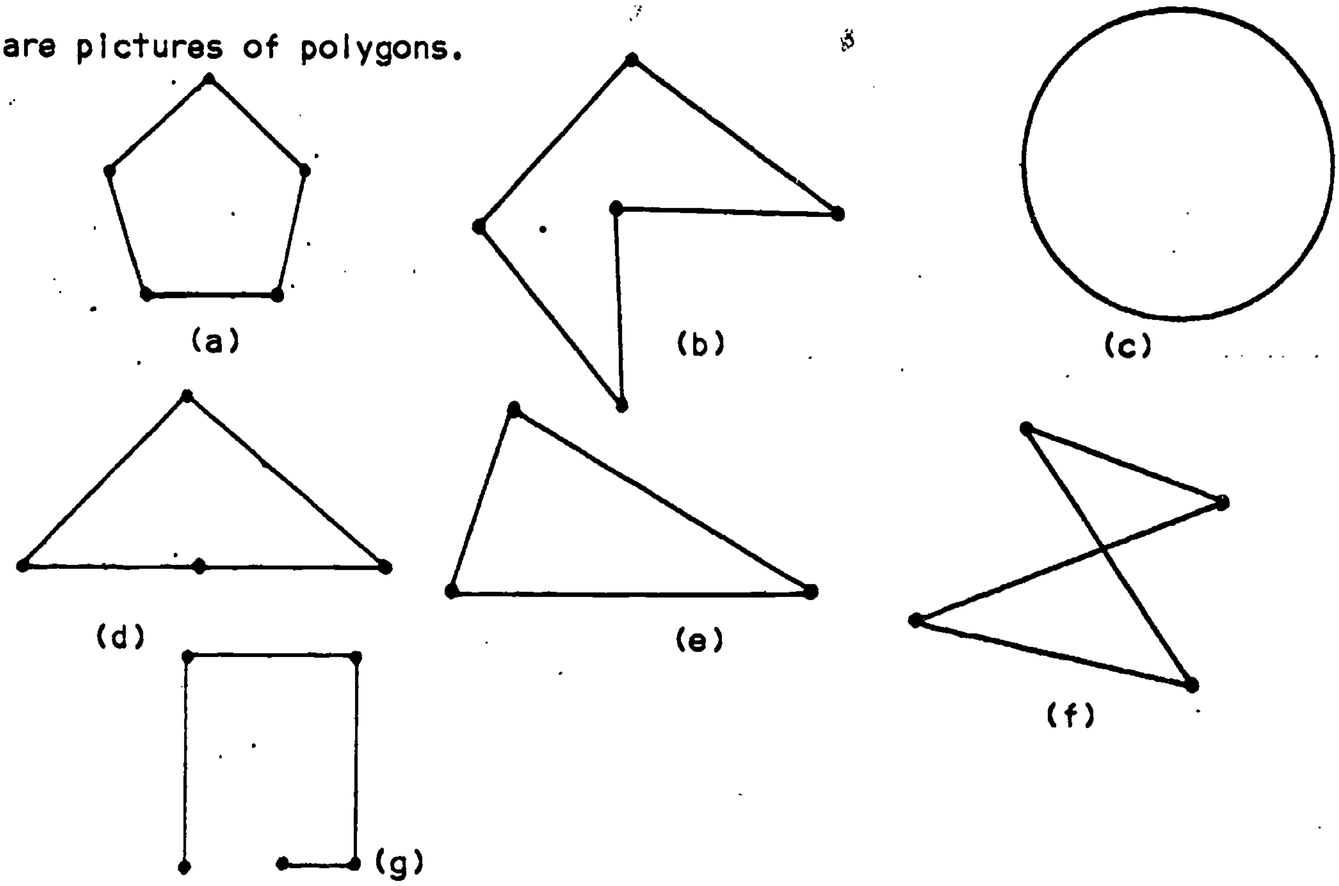
Section 6-8 Polygons and Polygonal Regions

In this section we are primarily interested in those simple closed paths that consist entirely of line segments placed end to end successively such that:

(a) No two segments cross each other except at an endpoint.

(b) No two segments with a common endpoint lie on the same line.

We call a path such as the one described above a simple polygon, or simply, a polygon. Your pupils should draw several simple closed paths, free-hand, so as not to be encumbered with rulers, measuring, etc. The emphasis should be on visualizing the figures rather than getting all the lines straight and the angles exact, etc. They should sketch some paths which are polygons and some that are not. You could draw some paths on the board like those below and ask your pupils which of these are pictures of polygons.



For those that are not polygons, you should have the pupil tell why they are not. For example, in the figures above, (c) is not a polygon because it does not consist of line segments; (d) is not a polygon because two of its line segments with the same endpoint lie on the same line; (f) is not a polygon because two of its line segments (sides) cross each other; and (g) is not a polygon because the path is not closed.

The pupils should do the cutting and tracing suggested in this section. If each cut is made along one line segment whose endpoints are on adjacent sides of the polygon, each new polygon will be convex and will have one more side than the previous one.

It is convenient to name polygons of 3, 4, 5, 6, 8, and 10 sides, but your pupils may have trouble remembering the names. They should use the names triangle and quadrilateral for polygons of three and four sides, respectively, but we often refer to a polygon with more than four sides as a "five-sided polygon", a "six-sided polygon", etc. Emphasis should not be placed on giving the polygons their exact names.

Be sure your pupils understand the difference between convex and concave polygons. We sometimes say a convex polygon is one in which all the vertices point <u>outward</u> or away from the interior of the polygon. A polygon that is not convex is one in which at least one vertex points <u>inward</u> or toward the interior of the polygon.

A polygonal region is a set of points consisting of the points that are in the <u>interior</u> of the polygon along with the points that are <u>on</u> the polygon.

<u>Answers to Exercise 6-8</u>

Student Text - Page 62

One easy way to subdivide a convex polygon into non-overlapping triangles is to begin at any vertex and draw all the diagonals from that vertex. This will not always work with a polygon that is not convex. However, any non-convex polygon can be subdivided into non-overlapping triangles by drawing certain of its diagonals.

Section 6-9 Circles

Most of your pupils may already be familiar with the methods for drawing circles and may have already used a compass to do so.

Emphasize that a circle is a set of points but that the center of the circle is not a point of the circle. The center is in the interior of the circle and is necessary in defining the circle, but it does not belong to the set we call the circle.

A circular region is the set of points in the interior of the circle along with the set of points on the circle.

A circular arc or simply an arc is a set of points and is part of a circle. Have your pupils draw several arcs of different sizes. Have them draw two arcs that intersect in only one point and then two arcs that intersect in two points.

Answers to Exercise 6-9

Student Text - Page 65

1. A circle is a simple closed path.
2. A circular arc is a simple open path.
3. If the radius is taken to be greater than one inch (greater than half the distance between the two points), the two semi-circular arcs with centers at A and B will intersect in two distinct points. If the radius is taken to be less than one inch (less than half the distance between the two points), the two semi-circular arcs will not intersect.
4. (a) The radius must be greater than 1 1/2 inches (greater than half the distance between the two points).

   (b) The radius must be equal to 1 1/2 inches.

   (c) The radius must be less than 1 1/2 inches.

Section 6-10 Pairs of Line Segments

This section and subsequent sections will deal with informal geometry. Up to this point the student has been learning vocabulary and basic concepts -- perhaps revising some ideas that he already knew. Now he will begin to use these new concepts and the new vocabulary to study geometric relations.

No attempt is made to build up geometry as a system of statements that can be proved from a few assumptions. Rather the student is encouraged to use paper folding, copying of line segments and similar methods to discover relations which exist between different line segments, angles and so on. He is taught to use a compass to construct line segments perpendicular to a given line segment, to construct parallels, and to bisect an angle or a line segment.

Our view is that geometry is rooted in experience and that at this stage the student should be taught to organize this experience for himself and not be presented with a ready-made system. We often ask the question "Why?" but we do not expect a long argument for an answer. We are satisfied when the student says something which shows understanding. An answer should be convincing to him and to the other students. It is important that the student gain confidence in the use of his own mind. If we ask for proofs of the obvious or require him to memorize proofs constructed by others which he cannot imagine himself thinking of, we destroy his confidence in his own mental powers.

If we have two line segments we can ask whether they can be made to fit exactly or whether one of them is larger than the other. This question can be answered by laying one segment along the other or by copying one segment and moving the copy so that a comparison can be made.

We can ask for a line segment which is congruent to a given one but has a given endpoint. We may also require the new line segment to have a definite direction, that is, lie along a given ray drawn from this endpoint. These problems can be solved by using a copy of the given line segment. They can also be solved by using a compass.

It should be noted that nowhere in these sections on construction do we compare line segments by measuring them with a ruler. The idea of measurement is postponed until Chapter 10. At this point we can say that a certain line segment is longer than another but we cannot say how many times longer it is. We can arrange segments in order of length but we cannot as yet give a number to the length of a segment by choosing a <u>unit</u> of measure. Comparison comes before measurement, both historically and logically.

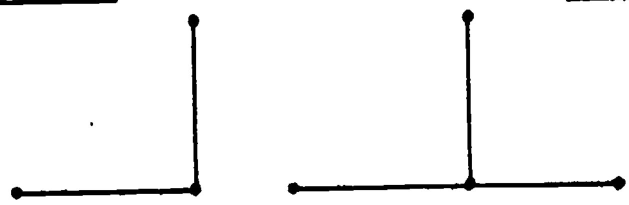
<u>Section 6-11 Perpendicular Bisector</u>

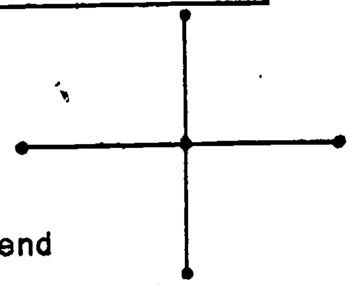
We learn how to divide a line segment into two congruent pieces. By paper folding and also by using compasses we discover how to find the midpoint of the line segment. These constructions give a line segment that not only bisect the given one but makes right angles with it. That is, we get not only a bisector but a perpendicular bisector. Of course, we do not allow ourselves to use a ruler. However, it is worth mentioning that we can bisect a line segment more accurately by paper folding than by the use of a ruler.

Student Text - Page 73

Answers to Exercise 6-11

2. 1, 2, or 4.





3. (a) crossing
   (b) connected end to end
   (c) touching
   (d) lined up end to end
   (e) non-intersecting

4. The line segments in (b) are congruent. None of the other pairs are congruent.

5. $\overline{OP}$ is the longest; $\overline{QR}$ is the shortest.

6. $\overline{PR}$ is the perpendicular bisector of $\overline{XY}$.
   $\overline{XY}$ is the perpendicular bisector of $\overline{DE}$, $\overline{PR}$, and $\overline{ST}$; and $\overline{FG}$ has no perpendicular bisector in the figure.

9. $\overline{HC}$, $\overline{SM}$, $\overline{GX}$, and $\overline{RN}$ are not congruent to any of the others.
   $\overline{PJ}$, $\overline{AD}$, and $\overline{WQ}$ are congruent to each other.

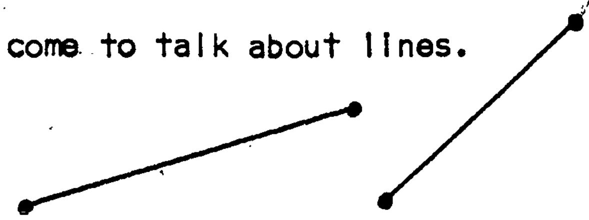
10. Listed from shortest to longest: $\overline{HG}$, $\overline{EF}$, $\overline{AB}$, $\overline{CD}$, $\overline{IJ}$.

11. The two line segments are perpendicular. The midpoint of $\overline{AB}$ is to the left of the point of intersection.

Section 6-12 Pairs of Lines

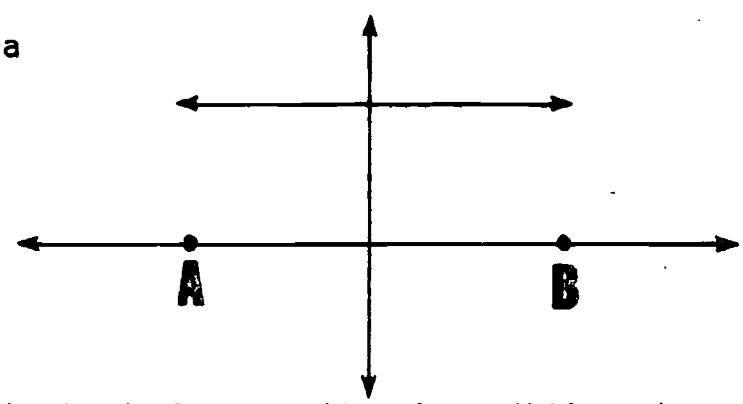
Two line segments in a plane may or may not intersect. But if they do not intersect, this may be because they are too short. If they are extended, the resulting segments may very well intersect.

For example, this is true in the figure shown. We are naturally led to consider the result of extending a line segment both ways without end. In this way we come to talk about lines.



We cannot draw lines. We can only draw segments of lines. In practice we draw sufficiently long segments to show the intersections in which we are interested. It is worth noticing that even though we cannot <u>draw</u> a whole line, the idea of a line is useful because it makes things simpler.

It may happen that two line segments in a plane are so placed that no matter how far we extend them there will be no intersection. We then say that the <u>lines</u> to which they belong are <u>parallel</u>. This is fine, but we need a way to decide whether two given lines are parallel or not. For example, if we have a line AB and a point P not on it, how can we draw a line through P so that we are sure that it will not intersect $\overleftrightarrow{AB}$?



The way this problem is met is to define parallel in a different way. <u>Two lines in a plane are said to be parallel if there is a line to which each of them is perpendicular</u>. This gives us a practical way of solving the problem because there is a simple way of drawing a line $\overleftrightarrow{PC}$ through P perpendicular to $\overleftrightarrow{AB}$ and there is a simple way of drawing a line through P perpendicular to $\overleftrightarrow{PC}$. These constructions are described in this section. Now, if two lines are parallel in the same sense that they lie in the same plane and are each perpendicular to a certain line, it is a fact that the lines are parallel in the same sense that they do not intersect. This is stated in Problem 3 but not proved.

It will be of interest to explain how this statement might be justified. Let $\overleftrightarrow{PD}$ and $\overleftrightarrow{AB}$ each be perpendicular to $\overleftrightarrow{PC}$. (See Figure 1 below.) If $\overleftrightarrow{PD}$ and $\overleftrightarrow{AB}$ did meet off the paper on one side of $\overleftrightarrow{PC}$, then by folding a larger piece of paper over $\overleftrightarrow{PC}$ there should be corresponding intersection on the other side because the figure is completely symmetrical about $\overleftrightarrow{PC}$. But then we would have two lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{PD}$ intersecting in two different points. This we regard as impossible.

We have not included this argument in the text, but it could be given if an alert student raises a question.

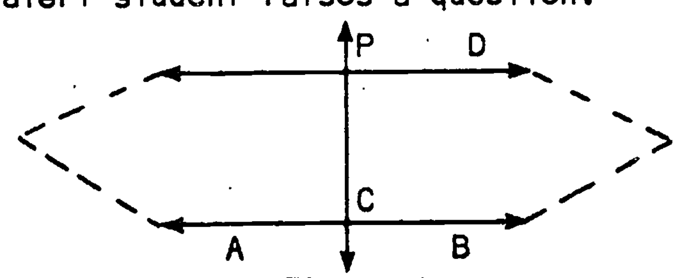


Figure 1

Answers to Exercise 6-12

Student Text - Page 86

1. (a) perpendicular
   (b) parallel
   (c) perpendicular
   (d) neither
   (e) parallel

2. (a), (c), and (d)

3. (a) No, for perpendicular lines form a right angle and therefore must intersect.

   (b) No, for parallel lines may never intersect.

4. $\overleftrightarrow{BT}$ and $\overleftrightarrow{IH}$ are perpendicular.

   The corners look like right angles.

   Right angles.

   $\overleftrightarrow{BF}$ is congruent to $\overleftrightarrow{TP}$, the corner at B fits the corner at T, and the corner at F fits the corner at P.

   The new line and $\overleftrightarrow{TP}$ are perpendicular, the new line and $\overleftrightarrow{PF}$ are parallel, and the new line and BT are parallel.

   (a $\overleftrightarrow{AI}$; $\overleftrightarrow{CD}$, $\overleftrightarrow{EF}$ and $\overleftrightarrow{GH}$ are perpendicular to $\overleftrightarrow{IH}$, and are also perpendicular to $\overleftrightarrow{LK}$.

   (b) $\overleftrightarrow{AI}$, $\overleftrightarrow{CD}$, $\overleftrightarrow{EF}$ and $\overleftrightarrow{GH}$ are all parallel to each other, and $\overleftrightarrow{IH}$ and $\overleftrightarrow{LK}$ are also parallel.

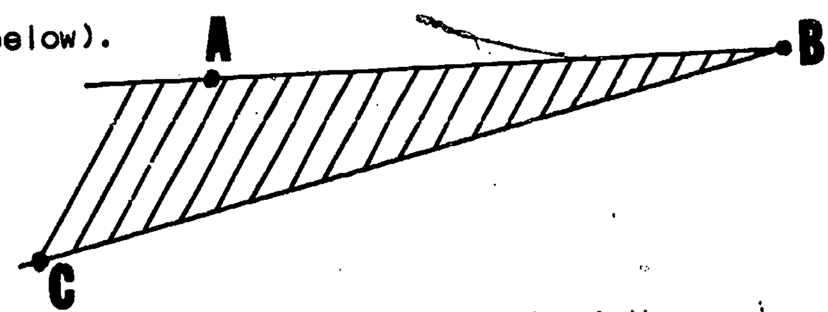
6. Yes, it is true because then all three of the lines can be shown to be perpendicular to the same line, and, therefore, are all parallel to each other.

<u>Section 6-13 Rays and Angles</u>

An angle is a figure which consists of two rays with a common endpoint. A right angle and a straight angle are important special cases. In this book, however, as is pointed out in the Student Text, two rays that have their endpoints in common so that the endpoint is between the other points of the two rays are called opposite rays (straight angle). At this point we also wish to be able to consider the notion of the <u>interior</u> of an angle. The notion of the interior of a straight angle might be somewhat ambiguous to the student at this stage and, hence, we have felt that it might be best to consider only angles less than a straight angle at this time.

We are also excluding angles greater than a straight angle.

The student is introduced to the notation which will be used for an angle and to the idea of the interior of an angle (the shaded region in the figure below).



Some definitions of angle permit us to speak of the angle outside the shaded region. We shall not do this.

It should be noted that measurement of angles in terms of a unit, for example, the degree, is not discussed in this chapter but is postponed until Chapter 10.

<u>Answers to Exercise 6-13</u>

Student Text - Page 92

1. (e) is impossible because, if the rays intersect at two points, they must lie along one line of sight and, therefore, must have many points in common.

   (h) is impossible because to make an angle they must have their endpoints in common.

2. Three angles can be named from the three rays: $\angle$ PJS, $\angle$ FJS and $\angle$ FJP. There are two right angles: $\angle$ PJS and $\angle$ FJS.

3. Four right angles are made by the rays and two straight angles.

4. (a) eight

Section 6-14 Comparing Angles

Earlier we considered comparing line segments so that two segments can be said to be congruent or, alternatively, one segment can be said to be larger than another. Similarly, we can compare angles. Either two angles are congruent to each other or one angle is larger than another. We decide which is true by fitting one ray of one angle exactly to a ray of the other so that the angle interiors overlap. If the other two rays fit exactly, the angles are said to be congruent. If the remaining ray of one angle lies in the interior of the other angle, the first angle is smaller than the second.

Answers to Exercise 6-14

Student Text - Page 96

1. $\angle$ IHB is larger.

Section 6-15 Angles Made by Lines

When two lines intersect they make four angles that are congruent in pairs. Vertically opposite angles are congruent. The student is expected to see this from the picture. No proof is required.

Strictly speaking, intersecting line segments do not make angles with each other. However, since they determine pairs of rays which are angles, we agree to call these pairs of rays the angles which the line segments make with each other. Similarly, by extending line segments into lines we can talk about parallel line segments when we mean that they are parts of parallel lines.

The method of naming angles used in the first figure in the section is a convenient one, but it should not be misunderstood. The numbers do not label the interiors of the angles. The interior of an angle is <u>not</u> part of the angle. The angle is the figure consisting of the rays. The use of the arrows pointing to the rays should help to keep this clear. Later these arrows may be omitted if there is no danger of misunderstanding.

Answers to Exercise 6-15

Student Text - Page 98

1. Two rays should be parallel if each of them is a part of one of two distinct parallel lines.

   They are parts of parallel lines.

   They are parts of parallel lines.

2. $\angle$RCB and $\angle$FCH are vertically opposite.
   $\angle$RCF and $\angle$BCH are vertically opposite.

3. (a) 1, Campbell and Hill are perpendicular to Lake Drive.

   2 is perpendicular to Hill.

   2 is parallel to Lake Drive.

   1 is parallel to Hill and Campbell.

   (b) At each intersection the vertically opposite angles are congruent. That is, vertically opposite corners of the street match each other.

5. Other examples are the sides of the door, parallel to each other and perpendicular to the top and bottom edges; the edges of a wall, parallel to each other and perpendicular to the base and top edge; and the side edges of a page in a book, parallel to each other and perpendicular to the top and bottom edges.

Section 6-16 Alternate Interior Angles

When two parallel lines are cut by a transversal, the pairs of alternate interior angles are congruent. This important fact is made plausible by paper folding.

In the problems the student combines this result with his knowledge that vertically opposite angles are congruent to find other pairs of congruent angles in the figure made by two parallel lines and a transversal.

It is a fact not mentioned here that if two lines are cut by a transversal so that alternate interior angles are equal, then the two lines are parallel.

Answers to Exercise 6-16 | Student Text - Page 102

1. (a) $\angle$ SBQ, $\angle$ BQC and $\angle$ NQT

   (b) $\angle$ HBS, $\angle$ RBQ and $\angle$ CQT

$\therefore$ $\angle$'s 1, 5, 6, 7, and 9 are all congruent.

$\angle$'s 2, 3, and 4 are congruent.

Section 6-17 Using a Compass to Compare Angles

In Section 6-14 we compared two angles directly by trying to fit one to the other. In Section 6-16 we showed by paper folding that certain angles are congruent. In this section we can compare two angles by making a copy of one of them and using the copy to see if it fits the second angle or not. A similar method was used to compare two line segments. To carry it out with angles we must know how to copy an angle. A way to do this with compasses is described.

Answers to Exercise 6-17 | Student Text - Page 106

1. b is larger.

2. Probably in this order: $\angle$BAD, $\angle$BAE, $\angle$BAC, $\angle$IHJ, $\angle$CAE, $\angle$DAC, $\angle$EAD, $\angle$KFG.

3. $\angle$ BAC is very slightly larger.

4. Yes, it shows that they match exactly.

5. They are all the same size.

Section 6-18 Bisecting an Angle

We learned how to divide a line segment into two adjacent congruent segments. We now learn to bisect a given angle, that is, to divide the angle into two adjacent congruent angles. The construction with compasses is simple but important.

Answers to Exercise 6-18 | Student Text - Page 110

1. Fold the paper along $\overrightarrow{TB}$. If $\overrightarrow{TA}$ and $\overrightarrow{TC}$ coincide, then the angles are congruent. If they are not congruent, then the smaller angle will be seen to fall entirely inside the larger. That is, the second side of the smaller angle will be contained in the interior of the larger.

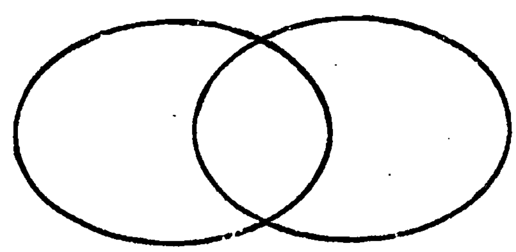
3. Changing the setting will not change the fact that the intersection of the arcs lies on the bisector of the angle. The pattern revealed by the intersections of the arcs is that they all lie

on one line, as long as the same setting is used for the two arcs for one intersection. It does not matter whether the same setting is used for the first large arc.

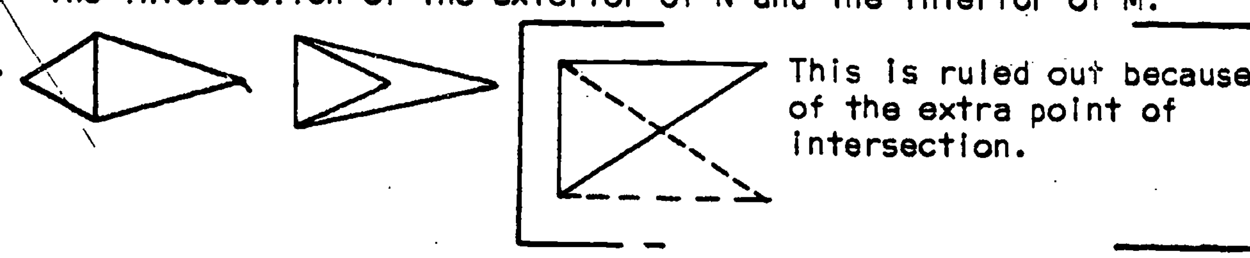
Chapter Review Answers

Student Text - Page 111

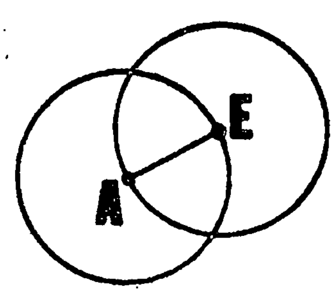
1. (a) Many (b) One
2. (a) Many (b) Many (c) One and only one unless the points are on a straight line.
3. Six



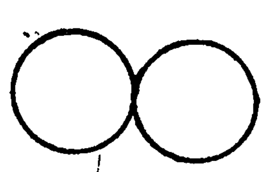
4. The intersection of the exterior of N and the interior of M.
5.



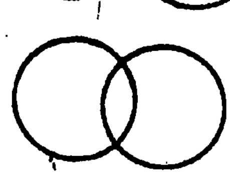
6. (c) The radius is not a part of the circle. The circle is the set of points that are a certain distance from the center.
   (d) No. Same as (c).
7. Only one.



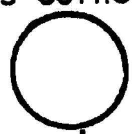
8. (a) No
   (b) Yes
   (c) Yes



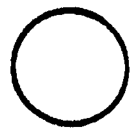
   (d) Yes



(e) Yes (Circles coincide.)



(f) Yes (Circles coincide.)



9. Copies of triangles.

10. The line segments in part (d) cannot be used to construct a triangle since the sum of the measures of any two sides does not exceed the measure of the third.

An attempted construction would look like this:

11. Answers will vary.

12. Answers will vary.

Chapter 7:

Factors and Primes

Teacher Commentary

This chapter gives the students considerable practice with multiplication and division of natural numbers. Explain to the students that immediate applications of GCF and LCM appear in the study of fractions. When "reducing a fraction to lowest terms," one is really dividing numerator and denominator by the GCF of the numerator and denominator. When adding or subtracting fractions with different denominators, one tries to find the least common denominator. The least common denominator of two or more fractions is the least common multiple of the denominators of these fractions.

Finally, the study of primes and the Fundamental Theorem of Arithmetic are important tools with which one studies the properties of natural numbers. Students at this level can get only a small glimpse of the importance of these tools. But it is a beginning.

General Objectives

A. To introduce the elementary properties of factors and multiples of numbers.

B. To show the value of these properties in the development of the concepts of greatest common factor (GCF) and least common multiple (LCM).

C. To introduce students to the Fundamental Theorem of Arithmetic.

Behavioral Objectives

1. The student can distinguish between the set of natural numbers and the set of whole numbers.
2. Given a mathematical sentence such as $48 = 6 \cdot 8$, the student can identify and name the factors (divisors).
3. Given a composite number named by two digits, the students can determine two of its factors.
4. Given two numbers (one prime, one composite) between 10 and 50, the student can identify and name the composite number and the prime number.
5. To the question "Is the number one prime or composite?", the student can respond that it is neither.
6. Given two consecutive numbers between 20 and 100, the student can identify and name the odd number and the even number.
7. The student can write an even number between 20 and 100 as a sum of two primes.
8. The student can list the first ten prime numbers.
9. The students can write the prime factorization of any composite number named by less than three digits.
10. Given three different composite numbers (each named by two digits), the student can determine the greatest common factor (GCF) of the numbers.
11. Given two numbers (one prime, one composite) between 1 and 20, the student can determine the lowest common multiple (LCM) of the numbers.

Section 7-1 Natural Numbers and Whole Numbers

The objectives of this section are (1) further introduction of the set of natural numbers and how it differs from the set of whole numbers, and (2) to make students fully realize that there are infinitely many natural numbers. One could try other examples in exercise 7. For example, {all people now alive}, {all grains of sand on all beaches in the world}. Both sets have fewer members than {natural numbers}.

Answers to Exercise 7-1

Student Text - Page 114

1. The set of natural numbers does not have 0 as a member.

2. N

3. W

4. I

5. 0

6. If there were a largest natural number, add 1 to it to get a number larger than the "largest number".

7. {natural numbers}. Since printing was not invented until the 15th century, one could probably prove easily that only a finite number of books have been published. Each book has only a finite number of pages. So the number of pages is finite.

8. No, w might represent 0.

Section 7-2 Factors and Divisors

Objectives and Comments:

(1) Introduce the definition of factor (divisor). Emphasize that a is a factor of n if (1) there exists a natural

number $\underline{b}$ such that $n = a \cdot b$, or (ii) when $\underline{n}$ is divided by $\underline{a}$, the quotient is some natural number $\underline{b}$, and the remainder is 0. For the set of natural numbers, (i) implies (ii), and (ii) implies (i). That is, if either condition (i) or (ii) is true, so is the other. If $n \div a = b$, then $n = a \cdot b$. If $n = a \cdot b$, then $n \div a = b$, with remainder 0. One word of caution about the definition: the critical point is not that there are only __two__ factors of n ($n = a \cdot b$). For example, $30 = 2(3 \cdot 5)$, $30 = 3(2 \cdot 5)$, $30 = 5(2 \cdot 3)$. We see that 2, 3, and 5 are each factors. The products within the grouping symbol can be thought of as one number.

(2) The word multiple is introduced later. We feel that it is better to introduce it when we are ready to use it.

Answers to Exercise 7-2(a)

Student Text - Page 116

1. 7
2. 8
3. In book.
4. and 5. a = 10, b = 3, (or a = 5, b = 6)
6. a. 1, 2, 4, 8
   b. 1, 2, 3, 4, 6, 12
   c. 1, 3, 5
   d. 1, 3, 7
   e. 1, 3, 9
   f. 1
7. a. 2, 10, 12, 44
   b. 6, 3, 21, 33, 15
   c. 12, 16, 28, 32
   d. 20, 30, 5, 60
   e. 24, 42, 6, 48
   f. 63, 28, 21, 14, 49
   g. 8, 56, 24, 16, 64
   h. 72, 45, 27, 36
   i. All numbers have 1 as a factor.

8. Answers can vary. 7, 11, and 41 are the only ones which cannot be written as products of 2 factors, each different from 1.

Discussion before Exercise 7-2(b) | Student Text - Page 118

The answers (a) through (i) are easy. You may wish to dwell on (e). The students should know that only numbers which end in 0 or 5 have 5 as a factor. There is a useful fact relating to (i) [not a fact that the students are expected to understand] which makes the finding of the factors easier. Namely, in looking for the set of factors of a number, one need try only numbers less than or equal to the square root of the number. For example, to find all the factors of 112, we need try only factors less than or equal to 11 (the smallest [natural] number whose square is greater than or equal to 112). For suppose $112 = a \cdot b$. It is not possible for both a and b to be greater than 11. Then $a \cdot b$ would be greater than 121. So if 112 can be factored as $a \cdot b$, one of a or b must be less than or equal to 11. It is, of course, possible that one of the factors of 112 is greater than 11. But such a factor will be found as the pair of a number less than 11.

Answers to Exercise 7-2(b) | Student Text - Page 119

1. natural number - a number which is a member of {1, 2, 3, 4, ...}.
   factor - a is a factor of n if (i) there exists b, a natural number, such that $n = a \cdot b$, or (ii) a divides into n b times with remainder 0.
   whole number - a number which is a member of {0, 1, 2, 3, ...}.
   divisor - same as factor.

C45

2.

Number	Set of Factors	Number of Factors	Number	Set of Factors	Number of Factors
1	{1}	1	13	{1, 13}	2
2	{1, 2}	2	14	{1, 2, 7, 14}	4
3	{1, 3}	2	15	{1, 3, 5, 15}	4
4	{1, 2, 4}	3	16	{1, 2, 4, 8, 16}	5
5	{1, 5}	2	17	{1, 17}	2
6	{1, 2, 3, 6}	4	18	{1, 2, 3, 6, 9, 18}	6
7	{1, 7}	2	19	{1, 19}	2
8	{1, 2, 4, 8}	4	20	{1, 2, 4, 5, 10, 20}	6
9	{1, 3, 9}	3	21	{1, 3, 7, 21}	4
10	{1, 2, 5, 10}	4	22	{1, 2, 11, 22}	4
11	{1, 11}	2	23	{1, 23}	2
12	{1, 2, 3, 4, 6, 12}	6	24	{1, 2, 3, 4, 6, 8, 12, 24}	8

3. The number of factors does not always increase as the natural numbers increase. Note 17 is larger than 12, but it has fewer factors. In fact, there are infinitely many numbers with exactly two factors.

4. 1 is a factor of every number.

5. {factors of 27} = {1, 3, 9, 27}

{factors of 30} = {1, 2, 3, 5, 6, 10, 15, 30}

{factors of 36} = {1, 2, 3, 4, 6, 9, 12, 18, 36}

{factors of 42} = {1, 2, 3, 6, 7, 14, 21, 42}

{factors of 48} = {1, 2, 3, 4, 6, 8, 12, 16, 24, 48}

Section 7-3 Prime Numbers and Composite Numbers

A prime number is sometimes defined as a number whose only factors are itself and 1. Some students then think 1 should be a prime number. They say that 1 has only itself and 1 as factors. The definition given in the text avoids this difficulty.

The 13 circles shown in the text suggest a possible class activity. Have the students try to divide a prime number of objects into groups with an equal number in each group. Use other primes in addition to 13. This same device could also be used on composite numbers to demonstrate all the different factors of a composite number. As a possible class activity, for example, start with a deck of cards (without the jokers). Ask the students what different groups of people (i.e., numbers in each group) could play with all cards dealt and no cards left over. Here, one is really asking the students to find all the factors of 52 (except 1 and 52). For example, 2 people could play with 26 cards, 4 people could play with 13 cards each. The set of factors of 52 is {1, 2, 4, 13, 26, 52}.

Regarding the Sieve Comments:

1. 1 is not considered. It is not a prime

2. Starting with 2, every other number has 2 as a factor.

   Starting with 3, every third number has 3 as a factor.

   Starting with 4, every fourth number has 4 as a factor.

   Starting with 5, every fifth number has 5 as a factor.

The reason for these patterns is clear. Think in terms of the remainders that occur when the numbers of the Sieve are divided by 2, 3, 4, 5, etc. We shall use 5 as an example: The remainder when 6, 7, 8, 9, and 10 are divided by 5 are 1, 2, 3, 4, and 0, respectively. The remainders when 11, 12, 13, 14, and 15 are divided by 5 are 1, 2, 3, 4, and 0, respectively. We see that every fifth number has 5 as a factor. This same reasoning applies to any number. Thus one can cross out composite numbers quickly.

The primes left by the Sieve are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

The word sieve is used in the sense of a "strainer", something which allows part of a mixture to pass through, catching the coarser part. Here, the primes pass through. The composites are caught by the sieve.

Answers to Exercise 7-3

Student Text - Page 124

1. Answers will vary.

   Sample answer:

   primes 2, 3, 5, 7, 11

   composites 4, 6, 8, 9, 10

   even 2, 4, 6, 8, 10

   odd 1, 3, 5, 7, 9

2. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 42, 47.

3. a. The empty set
   b. {natural ones}
   c. {3}
   d. {   }
   e. {all natural numbers but 1}
   f. {2}
   g. {1, 3, 5, 7, 9, ...} multiply by 3 to get {3, 9, 15, 21, 27, ...}
   h. {all primes except 2}

4. The phrase "It seems" is used here to avoid the habit of forming general rules when only a few examples are studied. "It seems" is not included in b. and f. For these, "proofs" are possible from definitions already available to the students. Actually, in every case, the answers given below are <u>always</u> true.

   a. <u>even</u> - There are easy proofs in elementary number theory. Perhaps it is sufficient to point out that when a number which ends in 1, 3, 5, 7, or 9 (odd numbers) is multiplied by a number ending in 0, 2, 4, 6, or 8, the product ends in 0, 2, 4, 6, or 8, making it an even number.

   b. never a <u>prime</u>. The product will have more than 2 factors.

   c. <u>even</u> - When a number which ends in 0, 2, 4, 6, or 8 is added to one which ends in 0, 2, 4, 6, or 8, the sum ends in 0, 2, 4, 6, or 8.

   d. <u>even</u> - When two numbers which end in 1, 3, 5, 7, or 9 are added together, the sum ends in 0, 2, 4, 6, or 8.

   e. <u>odd</u> - Reason as above.

   f. <u>odd</u> - 2 is the only even prime number.

5.

16 = 5 + 11	24 = 11 + 13	32 = 3 + 29	40 = 3 + 37	48 = 5 + 43
18 = 7 + 11	26 = 13 + 13	34 = 17 + 17	42 = 11 + 31	50 = 7 + 43
20 = 7 + 13	28 = 5 + 23	36 = 5 + 31	44 = 13 + 31	52 = 5 + 47
22 = 11 + 11	30 = 7 + 23	38 = 19 + 19	46 = 5 + 41	54 = 7 + 47

The list is given up to 54. In some cases there are other possibilities. Thus 26 = 3 + 23 = 7 + 19, as well as 13 + 13.

It certainly looks as if it is true that any even number greater than 4 can be expressed as the sum of two primes, but we surely have not proved it. No number of examples can ever be enough to make us sure that the result is always true. As a matter of fact, this is an example of a theorem, or rather a conjecture, which is simple to state but which is extremely difficult either to prove or to disprove. No one has yet (1969) been able to do so.

6. For example, 10 has 5 as a factor. 15 has 5 as a factor. 10 + 15 = 25, and 25 has 5 as a factor. The rule present here is always true: If a natural number c is a factor of a and a factor of b, then c is a factor of $a + b$.

Section 7-4 Prime Factorization of Natural Numbers

A method often used to factor numbers into prime factors is to try successively the primes 2, 3, 5, 7, 11, as shown below: Do not bother with composite numbers.

Factor 140 into prime factors: Study the algorithm given below. The partial quotients are blocked off. The primes are circled.

$$\begin{array}{r|l} 2 & 140 \\ 2 & 70 \\ 5 & 35 \\ 7 & 7 \\ & 1 \end{array}$$

$$140 = 2 \cdot 2 \cdot 5 \cdot 7$$

Answers to Exercise 7-4 — Student Text - Page 127

1. $15 = 3 \cdot 5$ $\quad 20 = 2 \cdot 2 \cdot 5$ $\quad 82 = 2 \cdot 41$

   $16 = 2 \cdot 2 \cdot 2 \cdot 2$ $\quad 36 = 2 \cdot 2 \cdot 3 \cdot 3$ $\quad 154 = 2 \cdot 7 \cdot 11$

   $18 = 2 \cdot 3 \cdot 3$ $\quad 48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$ $\quad 221 = 13 \cdot 17$

2. $9 = 3 \cdot 3$ $\quad 24 = 2 \cdot 2 \cdot 2 \cdot 3$ $\quad 108 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$

   $12 = 2 \cdot 2 \cdot 3$ $\quad 30 = 2 \cdot 3 \cdot 5$ $\quad 125 = 5 \cdot 5 \cdot 5$

   $21 = 3 \cdot 7$ $\quad 42 = 2 \cdot 3 \cdot 7$ $\quad 1{,}015 = 5 \cdot 7 \cdot 29$

3. $125 = 5 \cdot 5 \cdot 5$

$125 = 5 \cdot 5 \cdot 5$	number itself and 1	1, 125	factors
$5 \cdot 5 \cdot 5$	factors with 1 prime	5	
$5 \cdot 5 \cdot 5$	factors with 2 primes	25	

{factors of 125} = {1, 5, 25, 125}

$18 = 2 \cdot 3 \cdot 3$	number itself and 1	1, 18	
$2 \cdot 3 \cdot 3$	factors with 1 prime	2, 3	factors
$2 \cdot 3 \cdot 3$	factors with 2 primes	6, 9	

{factors of 18} = {1, 2, 3, 6, 9, 18}

$24 = 2 \cdot 2 \cdot 2 \cdot 3$	number itself and 1	1, 24	
$2 \cdot 2 \cdot 2 \cdot 3$	factors with 1 prime	2, 3	factors
$2 \cdot 2 \cdot 2 \cdot 3$	factors with 2 primes	4, 6	
$2 \cdot 2 \cdot 2 \cdot 3$	factors with 3 primes	8, 12	

{factors of 24} = {1, 2, 3, 4, 6, 8, 12, 24}

$108 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$	number itself and 1	1, 108	
$2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$	factors with 1 prime	2, 3	
$2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$	factors with 2 primes	4, 6, 9	factors
$2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$	factors with 3 primes	12, 18, 27	
$2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$	factors with 4 primes	36, 54	

{factors of 108} = {1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108}

$1{,}015 = 5 \cdot 7 \cdot 29$	number itself and 1	1, 1,015	
$5 \cdot 7 \cdot 29$	factors with 1 prime	5, 7, 29	factors
$5 \cdot 7 \cdot 29$	factors with 2 primes	35, 203, 145	

{factors of 1,015} = {1, 5, 7, 29, 35, 145, 203, 1,015}

Answers to Exercise 7-5

Student Text - Page 129

1. a = 2 2. b = 9, c = 3 3. d = 2

4. e = 2 5. f = 9 6. g = 3, h = 9

7.

	42	
6		7
2 3		7

Try also 2 • 21

and 3 • 14

8.

	36	
4		9
2 2		3 3

Try also 2 • 18

and 3 • 12

and 6 • 6

9.

	60	
2		30
2	6	5
2 2	3	5

Try also 3 • 20

and 4 • 15

and 5 • 12

and 6 • 10

10. (a) 2 • 5 (b) 3 • 5 (c) 3 • 3

(d) 2 • 2 • 5 • 5 (e) 2 • 2 • 7 (f) 2 • 2 • 2 • 2

(g) 2 • 2 • 2 • 3 • 3 (h) 3 • 3 • 3 • 3 (i) 3 • 5 • 5

Section 7-6 Greatest Common Factor

Care should be taken to avoid confusion between the terms Greatest Common Factor and Least Common Multiple. It is important that the students be thoroughly familiar with GCF (Section 7-6) before LCM (Section 7-7) is introduced.

The Fundamental Theorem of Arithmetic can be used to find the GCF of two or more numbers quickly. The GCF can be found by taking the product of the common prime factors. If there are no common prime factors, the GCF is 1.

Examples: Find the GCF of:

$12 = 2 \cdot 2 \cdot 3$ GCF of 12 and 30 =

$30 = 2 \cdot 3 \cdot 5$ $\underline{2 \cdot 3} = 6$

$20 = 2 \cdot 2 \cdot 5$ GCF = 1.

$27 = 3 \cdot 3 \cdot 3$ No common prime factors

$24 = 2 \cdot 2 \cdot 2 \cdot 3$ GCF = $2 \cdot 2 \cdot 3$

$108 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$ = 12

Answers to Exercise 7-6

Student Text - Page 131

1. a. {1, 2, 3, 6} b. {1, 2, 4, 8}
   c. {1, 2, 3, 4, 6, 12} d. {1, 3, 5, 15}
   e. {1, 2, 4, 8, 16} f. {1, 3, 7, 21}

2. a. {factors of 6} ∩ {factors of 8} = {1, 2, 3, 6} ∩ {1, 2, 4, 8} = {1, 2}. GCF = 2.

b. GCF = 4

c. GCF = 3

d. GCF = 2

e. GCF = 3

f. GCF = 4

3. GCF = 5

4. GCF = 6

5. GCF = 12

6. GCF = 25

7. GCF = 16

8. GCF = 3

9. GCF = 12

10. GCF = 15

Section 7-7 Multiples, Common Multiples and LCM

We shall use the Fundamental Theorem to find the LCM (Least Common Multiple) as we did with the GCF (Greatest Common Factor).

Be certain to emphasize the relationships between the words factor and multiple. Also, point out that the complete set of multiples of a number can be found by multiplying each natural number by the number being considered.

When referring to "columns" and "rows" of the multiplication table, be consistent. The columns are "vertical", and the rows are "horizontal".

Answers to Exercise 7-7

Student Text - Page 135

1. a. 2, 4, 6, 8, ..., 24

   b. 3, 6, 9, 12, ..., 36

   c. through h. - easy

2. a. 6, 12, 18, 24, 30, 36, 42

   b. 6, 12, 18, 24, 30, 36, 42

   c. 20, 40

   d. 15, 30

   e. 40

   f. 18, 36

3. a. $LCM = 6 = a \cdot b$ b. $LCM = 6 \neq a \cdot b = 18$

c. $LCM = 20 = a \cdot b$ d. $LCM = 24 \neq a \cdot b = 48$

e. $LCM = 15 = a \cdot b$ f. $LCM = 30 = a \cdot b$

4. a. {multiples of 2} = {2, 4, 6, 8, 10, 12, ...}

{multiples of 3} = {3, 6, 9, 12, 15, ...}

{multiples of 4} = {4, 8, 12, 16, ...}

LCM = 12

b. {multiples of 8} = {8, 16, 24, 32, 40, 48, 56, 64, 72, ...}

{multiples of 9} = {9, 18, 27, 36, 45, 54, 63, 72, ...}

{multiples of 12} = {12, 24, 36, 48, 60, 72, ...}

LCM = 72. There is no easy way to decide how many multiples to include in each set before a common multiple appears. This technique will be replaced by an easier method.

c. LCM = 60

d. LCM = 12

Answers to Exercise 7-8

Student Text - Page 139

1. Answered in text.

2. $14 = 2 \cdot 7$

$18 = 2 \cdot 3 \cdot 3$

$LCM = 14 \cdot a$

$LCM = 18 \cdot b$

Since the LCM is a multiple of 14 and a multiple of 18, we know that where a and b are natural numbers. (This is the definition of LCM.)

LCM = 2 • 7 • a

LCM = 2 • 3 • 3 • b

Then in the prime factorization of the LCM we need 1 factor of 2, 1 factor of 7, and 2 factors of 3. Thus

<u>LCM = 2 • 3 • 3 • 7 = 126</u>

3. 10 = 2 • 5

14 = 2 • 7

<u>LCM = 2 • 5 • 7 = 70</u>

4. 16 = 2 • 2 • 2 • 2

18 = 2 • 3 • 3

LCM = 2 • 2 • 2 • 2 • 3 • 3

LCM = 16 • 9

<u>LCM = 144</u>

5. 12 = 2 • 2 • 3

17 = 17 • 1 (a prime)

LCM = 2 • 2 • 3 • 17

LCM = 12 • 17

<u>LCM = 204</u>

6. $60 = 2 \cdot 2 \cdot 3 \cdot 5$

$36 = 2 \cdot 2 \cdot 3 \cdot 3$

$LCM = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$

$\underline{LCM = 180}$

7. $100 = 2 \cdot 2 \cdot 5 \cdot 5$

$250 = 2 \cdot 5 \cdot 5 \cdot 5$

$200 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5$

$LCM = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$

$\underline{LCM = 1000}$

Answers to Chapter Review

Student Text - Page 140

1. a. a, b, c, e
   b. a, b, d, f
   c. a, b, d
   d. a, b, d, e
   e. a, b, c, f
   f. a, c?

In f., c has a question mark next to it. Ordinarily, 0 is considered an even number by mathematicians, since $0 = 2 \cdot 0$. In this text we have not considered 0 in our discussion of factors and multiples.

2. a. already done
   b. already done
   c. 2, 7, 14
   d. 2, 4, 8, 16 (any 3)
   e. 2, 3, 4, 6, 8, 12, 24 (any 3)
   f. 3, 9, 27
   g. 2, 4, 8, 16, 32 (any 3)

3. a. already done
   b. already done
   c. 14, 21, 28, etc.
   d. 24, 36, 48, etc.
   e. 18, 27, 36, etc.
   f. 10, 15, 20, etc.

4. 15 primes less than 50: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47.

5. 5, 10, 15, ..., 55, 60

6. 7, 14, 21, 28, 35, 42, 49

7. 15, 30, 45, 60, 75, 90

8. a. {common factors of 18 and 42} = {1, 2, 3, 6}
   b. {common factors of 21 and 33} = {1, 3}

9. a. Use 8a or find common primes

   $18 = 2 \cdot 3 \cdot 3$

   $42 = 2 \cdot 3 \cdot 7$

   GCF = 6

   b. $28 = 2 \cdot 2 \cdot 7$

   $56 = 2 \cdot 2 \cdot 2 \cdot 7$

   $GCF = 2 \cdot 2 \cdot 7 = 28$

10. a. $8 = 2 \cdot 2 \cdot 2$

   $10 = 2 \cdot 5$

   $LCM = 2 \cdot 2 \cdot 2 \cdot 5 = \underline{40}$

   b. $12 = 2 \cdot 2 \cdot 3$

   $15 = 3 \cdot 5$

   $LCM = 2 \cdot 2 \cdot 3 \cdot 5 = \underline{60}$

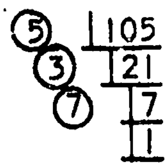
   c. $10 = 2 \cdot 5$

   $15 = 3 \cdot 5$

   $30 = 2 \cdot 3 \cdot 5$

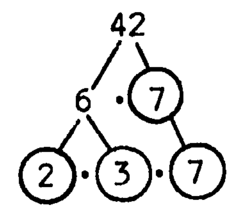
   $LCM = 2 \cdot 3 \cdot 5 = \underline{30}$

11. a.



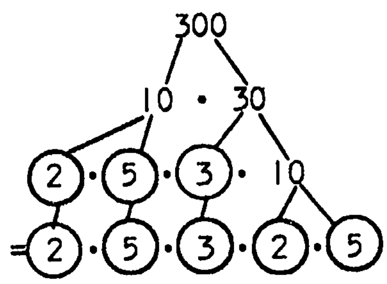
$105 = 5 \cdot 3 \cdot 7$

b.

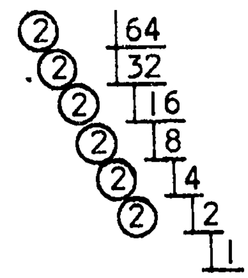


$42 = 2 \cdot 3 \cdot 7$

c.



d.



$64 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$= 2^6$

12. a. GCF 3, 5 = 1 $\qquad a \cdot b = 15$

LCM 3, 5 = 15 $\qquad G \cdot L = 15$

b. GCF 8, 20 = 4 $\qquad a \cdot b = 160$

LCM 8, 20 = 40 $\qquad G \cdot L = 160$

13. a. $5 + 1 + 0 + 2 + 1 + 1 = 10$. The number <u>does not</u> have 9 as a factor since 10 does not.

b. $2 + 1 + 1 + 5 = 9$. The number <u>does</u> have 9 as a factor, since 9 is a factor of itself.

c. $510{,}211 = (9 \cdot 56690) + 1$

$2{,}115 = 9 \cdot 235$

14. a. $b = 10$, $c = 100$ $\qquad$ 5 is a factor of 10, 10 is a factor of 100, so 5 is a factor of 100.

b. $c = 42$ $\qquad$ 14 divides 42, then 7 divides 42.

15. Find GCF's in any way.

16. 4 does not divide 176,930 since 4 does not divide 30, the number formed by the last two digits.

4 does divide 624, since 4 divides 24.